

RESEARCH ARTICLE

Electroosmotic peristaltic transport of Williamson fluid through a flexible annular channel

A. I. Abdellateef¹  and Syed Zahoor Ul Haque^{2,*} 

¹*Department of Applied Mathematics and Science, Faculty of Engineering, National University of Science and Technology, AlSeeb 111, Sultanate of Oman.*

²*Department of Mathematics and IT, Sultan Qaboos University, Alkhoud, Sultanate of Oman.*

**Corresponding Author. Email: z.syed@squ.edu.om (Z. S. Ul Haque)*

Article Information

Received: 20 July 2026

Accepted: 13 October 2026

Published 21 October 2026

AMS Classification:

76A05, 76M45, 76D08

Abstract

The present work develops an analytical model for electroosmotic peristaltic transport of a Williamson fluid in a flexible annular channel. The flow takes place between two concentric cylinders, where the inner wall is rigid and the outer wall undergoes sinusoidal elastic deformation. An externally applied axial electric field interacts with the electric double layer generated by prescribed wall zeta potentials. The electric potential is governed by the Poisson–Boltzmann equation and is linearized using the Debye–Huckel approximation. Under the assumptions of long wavelength and low Reynolds number, the governing equations are reduced to a nonlinear ordinary differential equation for the axial velocity. A regular perturbation expansion in the Weissenberg number is employed to obtain analytical expressions for the velocity, shear stress, and stream function. The model shows that electroosmotic forcing, wall zeta potential, annular geometry, wall elasticity, and Williamson rheology significantly influence the velocity field, wall shear stress, and trapping pattern. In the absence of electroosmotic forcing, the formulation reduces to the classical flexible annular Williamson-fluid peristaltic model.

Keywords: Williamson fluid, electroosmosis, peristaltic flow, annular channel, flexible wall, perturbation method

1. Introduction

Peristaltic transport is produced by progressive waves of contraction and expansion along deformable boundaries. It plays a central role in several physiological processes, including chyme transport in the gastrointestinal tract, urine movement from the kidney to the bladder, oesophageal swallowing, blood motion in small vessels, and embryo transport. It is also relevant to engineering systems such as roller pumps, heart–lung machines, biomedical devices, and the pumping of corrosive or hazardous fluids without direct mechanical contact. Because of these applications, peristaltic flow has been studied in planar, cylindrical, porous, curved, and annular geometries for both Newtonian and non-Newtonian fluids [1–5]. The fundamental theory of peristaltic pumping was initiated by Latham [6]. Shapiro et al. [7] established the long-wavelength and low-Reynolds-number framework, while Yin and Fung [8] compared theoretical predictions with experimental observations. Later studies incorporated compliant walls, heat and mass transfer, porous media, and annular geometries. For example, Radha Krishnamacharya and

Srinivasulu [9] examined the influence of wall properties on peristaltic transport with heat transfer, Srinivas and Kothandapani [10] considered compliant wall effects in a porous channel, and Mekheimer et al. [11] investigated peristaltic transport through eccentric cylinders. Many biological and industrial fluids exhibit nonlinear rheological behaviour. Polymer solutions, suspensions, emulsions, and physiological fluids may display shear-thinning or viscoelastic features that cannot be described accurately by Newtonian models. The Williamson model is commonly used to describe pseudoplastic fluids because it accounts for the decrease in apparent viscosity with increasing shear rate. Hayat et al. [12] analysed peristaltic Williamson-fluid flow in an inclined channel with compliant walls, while Yasmin et al. [13] studied convective heat and mass transfer in Williamson-fluid peristalsis. Ellahi et al. [14] considered three-dimensional peristaltic motion of a Williamson fluid in a rectangular duct. Electroosmosis is an important electrokinetic mechanism in microfluidic and biomedical transport. It occurs when an externally applied electric field acts on the excess charge density within the electric double layer near charged walls. The resulting electric body force drags the surrounding liquid through viscous interaction and generates electroosmotic motion. Electroosmotic peristaltic transport has been examined in microchannels and biological configurations by Bhatti et al. [15], Prakash and Tripathi [16], Tripathi et al. [17], and Noreen et al. [18]. Recent studies have further emphasized the role of zeta potential, electrokinetic forcing, and non-Newtonian rheology in microchannel flows [19–23]. The present work extends the flexible annular-channel Williamson-fluid model of Al-Khafajy and Al-Delfi [24] by incorporating electroosmotic effects. The proposed model retains the cylindrical geometry, flexible-wall operator, wave-frame transformation, lubrication approximation, and perturbation approach, while introducing an axial electric field, electric-double-layer potential, and electroosmotic body force. The main objective is to establish a compact analytical framework for studying the combined effects of wall elasticity, electroosmosis, annular geometry, and Williamson rheology on peristaltic transport.

2. Mathematical formulation

Consider the axisymmetric peristaltic motion of an incompressible Williamson fluid through an annular region formed by two concentric cylinders. Cylindrical coordinates are denoted by $(\bar{r}, \bar{\vartheta}, \bar{z})$, where $\bar{r}$ is the radial coordinate and $\bar{z}$ is the axial coordinate. The inner cylinder is rigid with radius a_1 , while the outer wall is flexible and is described by a sinusoidal wave,

$$\bar{r} = a_1, \quad \text{inner wall}, \quad (1)$$

$$\bar{r} = \bar{r}_2(\bar{z}, \bar{t}) = a_2 + b \sin \left[\frac{2\pi}{\mathcal{L}}(\bar{z} - s\bar{t}) \right], \quad \text{outer wall}, \quad (2)$$

where a_2 is the mean radius of the outer cylinder, b is the wave amplitude, $\mathcal{L}$ is the wavelength, s is the wave speed, and $\bar{t}$ is time.

The velocity field is assumed to be

$$\bar{V} = (\bar{V}_1(\bar{r}, \bar{z}, \bar{t}), 0, \bar{V}_3(\bar{r}, \bar{z}, \bar{t})), \quad (3)$$

where $\bar{V}_1$ and $\bar{V}_3$ denote the radial and axial velocity components, respectively. An external electric field is applied in the axial direction,

$$E = (0, 0, E_z). \quad (4)$$

The charged walls generate an electric double layer. The dimensional electric potential and net charge density are denoted by $\bar{\Phi}(\bar{r})$ and $\bar{\rho}_e$, respectively.

The continuity and momentum equations are

$$\bar{\nabla} \cdot \bar{V} = 0, \quad (5)$$

$$\rho \left(\frac{\partial \bar{V}}{\partial \bar{t}} + \bar{V} \cdot \bar{\nabla} \bar{V} \right) = \bar{\nabla} \cdot (\bar{p}I + \bar{\tau}) + \bar{\rho}_e E, \quad (6)$$

where ρ is the fluid density, $\bar{p}$ is the pressure, I is the identity tensor, and $\bar{\tau}$ is the extra-stress tensor. The last term in Eq. (6) represents the electroosmotic body force.

The pressure convention used in the reduced balance is $-pI + \tau$, consistently with Eq. (22).

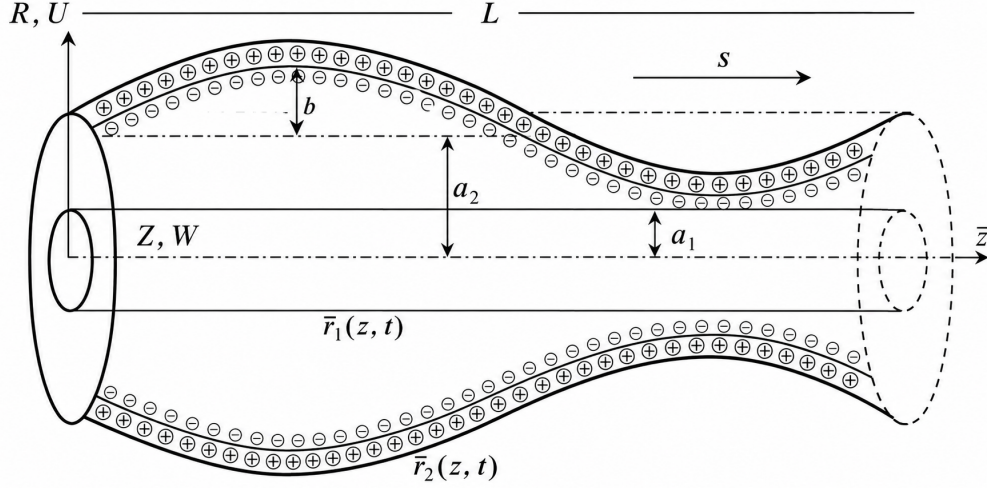


Figure 1. Geometry of the electroosmotic peristaltic flow through a flexible annular channel.

2.1. Williamson constitutive model

The Williamson constitutive relation is written as

$$\bar{\tau} = \left[\mu_\infty + \frac{\mu_0 - \mu_\infty}{1 - \Gamma \dot{\bar{\gamma}}} \right] \dot{\bar{\gamma}}, \quad (7)$$

where μ_0 and μ_∞ are the zero- and infinite-shear-rate viscosities, respectively. The parameter Γ is the Williamson time constant and $\dot{\bar{\gamma}}$ denotes the shear rate. For $\mu_\infty = 0$ and small $\Gamma \dot{\bar{\gamma}}$, the dominant shear-stress component under the lubrication approximation becomes

$$\bar{\tau}_{\bar{r}\bar{z}} = \mu_0 \left[\frac{\partial \bar{v}_3}{\partial \bar{r}} + \Gamma \left(\frac{\partial \bar{v}_3}{\partial \bar{r}} \right)^2 \right]. \quad (8)$$

For signed $y = \partial v_3 / \partial r$, the scalar closure in Eq. (23) is $y + We y^2$. The numerical check below tests this reduced closure; an invariant-based shear-rate formulation requires a consistent $y|y|$ convention across changes of shear direction.

2.2. Electric potential and charge density

The electric potential satisfies Poisson's equation,

$$\frac{1}{\bar{r}} \frac{\partial}{\partial \bar{r}} \left(\bar{r} \frac{\partial \bar{\Phi}}{\partial \bar{r}} \right) = -\frac{\bar{\rho}_e}{\epsilon_e}, \quad (9)$$

where ϵ_e is the electric permittivity. Under the Debye–Hückel approximation,

$$\bar{\rho}_e = -\epsilon_e \kappa^2 \bar{\Phi}, \quad (10)$$

where κ^{-1} is the Debye length. Hence,

$$\frac{1}{\bar{r}} \frac{\partial}{\partial \bar{r}} \left(\bar{r} \frac{\partial \bar{\Phi}}{\partial \bar{r}} \right) = \kappa^2 \bar{\Phi}. \quad (11)$$

The wall zeta-potential conditions are

$$\bar{\Phi}(a_1) = \bar{\zeta}_1, \quad \bar{\Phi}(\bar{r}_2) = \bar{\zeta}_2. \quad (12)$$

2.3. Flexible-wall relation

The flexible outer wall is modelled as a stretched membrane with flexural rigidity, longitudinal tension, inertia, viscous damping, and spring stiffness. The wall operator is

$$L^* = A \frac{\partial^4}{\partial \bar{z}^4} - B \frac{\partial^2}{\partial \bar{z}^2} + C \frac{\partial^2}{\partial \bar{t}^2} + D \frac{\partial}{\partial \bar{t}} + E_L, \quad (13)$$

where A , B , C , D , and E_L denote flexural rigidity, longitudinal tension per unit width, mass per unit area, viscous damping, and spring stiffness, respectively. At the flexible wall,

$$\frac{\partial \bar{p}}{\partial \bar{z}} = \frac{\partial}{\partial \bar{z}} \left(A \frac{\partial^4}{\partial \bar{z}^4} - B \frac{\partial^2}{\partial \bar{z}^2} + C \frac{\partial^2}{\partial \bar{t}^2} + D \frac{\partial}{\partial \bar{t}} + E_L \right) \bar{r}_2. \quad (14)$$

In Eq. (26), wall time derivatives are evaluated in the laboratory frame before the transformation in Eq. (15); the wall operator acts on the outer-wall displacement. The signed coefficients in Eq. (27) are used as defined; positive plotted L_2 values are treated as formal coefficient variations.

3. Non-dimensionalities and reduced governing equations

The fixed frame is transformed to the wave frame using

$$\bar{r} = \bar{R}, \quad \bar{z} = \bar{Z} - s\bar{t}, \quad \bar{v}_1 = \bar{V}_1, \quad \bar{v}_3 = \bar{V}_3 - s. \quad (15)$$

In the wave frame, the dimensionless wall positions are

$$r_1 = \epsilon, \quad r_2 = 1 + \phi \sin(2\pi z), \quad (16)$$

where

$$\epsilon = a_1/a_2 \quad \text{and} \quad \phi = b/a_2.$$

The following non-dimensional variables and parameters are introduced:

$$\begin{aligned} r &= \frac{\bar{r}}{a_2}, \quad z = \frac{\bar{z}}{\mathcal{L}}, \quad t = \frac{s\bar{t}}{\mathcal{L}}, \quad v_1 = \frac{\mathcal{L}\bar{v}_1}{a_2 s}, \quad v_3 = \frac{\bar{v}_3}{s}, \\ p &= \frac{a_2^2 \bar{p}}{\mu_0 s \mathcal{L}}, \quad \tau = \frac{a_2 \bar{\tau}}{\mu_0 s}, \quad \Phi = \frac{\bar{\Phi}}{\zeta_0}, \quad \delta = \frac{a_2}{\mathcal{L}}, \quad Re = \frac{\rho s a_2}{\mu_0}, \\ We &= \frac{\Gamma s}{a_2}, \quad m_e = a_2 \kappa, \quad U_s = -\frac{\epsilon_e E_z \zeta_0}{\mu_0 s}. \end{aligned} \quad (17)$$

Here We is the Weissenberg number, m_e is the electroosmotic parameter, and U_s is the dimensionless Helmholtz–Smoluchowski velocity. The dimensionless electric-potential equation is

$$\frac{1}{r} \frac{d}{dr} \left(r \frac{d\Phi}{dr} \right) = m_e^2 \Phi, \quad (18)$$

with

$$\Phi(\epsilon) = \zeta_1, \quad \Phi(r_2) = \zeta_2, \quad (19)$$

where

$$\zeta_1 = \bar{\zeta}_1/\zeta_0 \quad \text{and} \quad \zeta_2 = \bar{\zeta}_2/\zeta_0.$$

Using the long-wavelength approximation ($\delta \ll 1$) and the low-Reynolds-number approximation ($Re \ll 1$), the governing equations reduce to

$$\frac{\partial v_1}{\partial r} + \frac{v_1}{r} + \frac{\partial v_3}{\partial z} = 0, \quad (20)$$

$$\frac{\partial p}{\partial r} = 0, \quad (21)$$

$$\frac{\partial p}{\partial z} = \frac{1}{r} \frac{\partial}{\partial r} (r \tau_{rz}) + U_s \frac{1}{r} \frac{d}{dr} \left(r \frac{d\Phi}{dr} \right), \quad (22)$$

where

$$\tau_{rz} = \frac{\partial v_3}{\partial r} + We \left(\frac{\partial v_3}{\partial r} \right)^2. \quad (23)$$

The no-slip conditions in the wave frame are

$$v_3(\epsilon) = -1, \quad v_3(r_2) = -1. \quad (24)$$

The dimensionless wall relation can be expressed as

$$\frac{\partial p}{\partial z} = K(z, t), \quad (25)$$

where

$$K = L_1 \frac{\partial^5 r_2}{\partial z^5} + L_2 \frac{\partial^3 r_2}{\partial z^3} + L_3 \frac{\partial^3 r_2}{\partial z \partial t^2} + L_4 \frac{\partial^2 r_2}{\partial z \partial t} + L_5 \frac{\partial r_2}{\partial z}, \quad (26)$$

and

$$L_1 = \frac{Aa_2^3}{\mu_0 s \mathcal{L}^5}, \quad L_2 = -\frac{Ba_2^3}{\mu_0 s \mathcal{L}^3}, \quad L_3 = \frac{Csa_2^3}{\mu_0 \mathcal{L}^3}, \quad L_4 = \frac{Da_2^3}{\mu_0 \mathcal{L}^2}, \quad (27)$$

$$L_5 = \frac{E_L a_2^3}{\mu_0 s \mathcal{L}}.$$

Combining Eqs. (18), (22), (23), and (25) yield the nonlinear axial-velocity equation

$$r \frac{\partial^2 v_3}{\partial r^2} + \frac{\partial v_3}{\partial r} + We \left(\frac{\partial v_3}{\partial r} \right)^2 + 2rWe \frac{\partial v_3}{\partial r} \quad (28)$$

$$\frac{\partial^2 v_3}{\partial r^2} = r(K - U_s m_e^2 \Phi).$$

The approximation regime is $\delta \ll 1$, $Re \ll 1$, $2\pi|\phi|\delta \ll 1$, and $0 < \epsilon < 1 - |\phi|$. Debye–Hückel linearization requires $|\nu|e \max |\bar{\zeta}_i|/(k_B T) \ll 1$, where ν is ion valence, e elementary charge, k_B the Boltzmann constant, T temperature, and $\bar{\zeta}_i$ dimensional wall potential. The perturbation additionally requires $\chi = We \max |\partial v_3 / \partial r| \ll 1$, normalized wall potentials and nominal We alone do not establish this regime.

4. Analytical solution

4.1. Electric-potential distribution

Eq. (18) can be written as

$$\frac{d^2 \Phi}{dr^2} + \frac{1}{r} \frac{d\Phi}{dr} - m_e^2 \Phi = 0. \quad (29)$$

For $m_e \neq 0$, its solution is

$$\Phi(r) = A_e I_0(m_e r) + B_e K_0(m_e r), \quad (30)$$

where I_0 and K_0 are modified Bessel functions of the first and second kinds, respectively. The constants are obtained from Eq. (19) as

$$\Delta_e = I_0(m_e r_2) K_0(m_e \epsilon) - I_0(m_e \epsilon) K_0(m_e r_2), \quad (31)$$

$$A_e = \frac{\zeta_2 K_0(m_e \epsilon) - \zeta_1 K_0(m_e r_2)}{\Delta_e}, \quad (32)$$

$$B_e = \frac{\zeta_1 I_0(m_e r_2) - \zeta_2 I_0(m_e \epsilon)}{\Delta_e}. \quad (33)$$

4.2. Perturbation solution for axial velocity

Because Eq. (28) is nonlinear, a regular perturbation expansion is introduced for small We :

$$v_3(r) = v_{03}(r) + We v_{13}(r) + We^2 v_{23}(r) + O(We^3). \quad (34)$$

The boundary conditions are

$$v_{03}(\epsilon) = -1, \quad v_{03}(r_2) = -1, \quad (35)$$

$$v_{13}(\epsilon) = 0, \quad v_{13}(r_2) = 0, \quad (36)$$

$$v_{23}(\epsilon) = 0, \quad v_{23}(r_2) = 0. \quad (37)$$

4.2.1. Zeroth-order solution

The zeroth-order equation is

$$r \frac{d^2 v_{03}}{dr^2} + \frac{dv_{03}}{dr} = r(K - U_s m_e^2 \Phi). \quad (38)$$

Using Eq. (18), the solution is

$$v_{03}(r) = \frac{K}{4}r^2 - U_s\Phi(r) + C_0 \ln r + D_0, \quad (39)$$

where

$$H = \ln\left(\frac{r_2}{\epsilon}\right), \quad (40)$$

$$C_0 = \frac{-\frac{K}{4}(r_2^2 - \epsilon^2) + U_s(\zeta_2 - \zeta_1)}{H}, \quad (41)$$

$$D_0 = -1 - \frac{K}{4}\epsilon^2 + U_s\zeta_1 - C_0 \ln \epsilon. \quad (42)$$

4.2.2. First-order solution

The first-order equation is

$$r \frac{d^2 v_{13}}{dr^2} + \frac{dv_{13}}{dr} = -(v'_{03})^2 - 2rv'_{03}v''_{03}, \quad (43)$$

where the prime denotes differentiation with respect to r . Let

$$S_1(r) = -(v'_{03})^2 - 2rv'_{03}v''_{03}. \quad (44)$$

Then the first-order velocity correction is

$$v_{13}(r) = \mathcal{J}[S_1](r) - \frac{\mathcal{J}[S_1](r_2)}{H} \ln\left(\frac{r}{\epsilon}\right), \quad (45)$$

where

$$\mathcal{J}[S](r) = \int_{\epsilon}^r \frac{1}{\xi} \left(\int_{\epsilon}^{\xi} S(\eta) d\eta \right) d\xi. \quad (46)$$

4.2.3. Second-order solution

The second-order equation is

$$r \frac{d^2 v_{23}}{dr^2} + \frac{dv_{23}}{dr} = -2v'_{03}v'_{13} - 2r(v'_{03}v''_{13} + v'_{13}v''_{03}). \quad (47)$$

Let

$$S_2(r) = -2v'_{03}v'_{13} - 2r(v'_{03}v''_{13} + v'_{13}v''_{03}). \quad (48)$$

Thus,

$$v_{23}(r) = \mathcal{J}[S_2](r) - \frac{\mathcal{J}[S_2](r_2)}{H} \ln\left(\frac{r}{\epsilon}\right). \quad (49)$$

An explicit equivalent of Eq. (49) is:

$$v_{23}(r) = J[S_2](r) - \left[\frac{J[S_2](r_2)}{H} \right] \ln\left(\frac{r}{\epsilon}\right) \quad (49a)$$

The approximate axial velocity is

$$v_3(r) = v_{03}(r) + We v_{13}(r) + We^2 v_{23}(r) + O(We^3). \quad (50)$$

4.3. Shear stress and stream function

The dimensionless shear stress is

$$\tau_{rz} = v'_3 + We(v'_3)^2. \quad (51)$$

Using Eq. (34),

$$\tau_{rz} = \tau_0 + We\tau_1 + We^2\tau_2 + O(We^3), \quad (52)$$

where

$$\tau_0 = v'_{03}, \quad (53)$$

$$\tau_1 = v'_{13} + (v'_{03})^2, \quad (54)$$

$$\tau_2 = v'_{23} + 2v'_{03}v'_{13}. \quad (55)$$

The stream function is defined by

$$v_1 = -\frac{1}{r} \frac{\partial \psi}{\partial z}, \quad v_3 = \frac{1}{r} \frac{\partial \psi}{\partial r}. \quad (56)$$

Taking $\psi(\epsilon, z, t) = 0$, the stream function is

$$\psi(r, z, t) = \int_{\epsilon}^r \xi v_3(\xi, z, t) d\xi. \quad (57)$$

Accordingly,

$$\begin{aligned} \psi &= \psi_0 + We\psi_1 + We^2\psi_2 + O(We^3), \\ \psi_j &= \int_{\epsilon}^r \xi v_{j3}(\xi, z, t) d\xi, \quad j = 0, 1, 2. \end{aligned} \quad (58)$$

4.4. Limiting cases

The present model contains several physically meaningful limiting cases. When $U_s = 0$, the electroosmotic body force vanishes, and Eq. (28) reduces to the non-electroosmotic flexible annular Williamson-fluid model. When $We = 0$, the model represents electroosmotic peristaltic motion of a Newtonian viscous fluid. When the wall parameters are suppressed, the formulation approaches the corresponding rigid-wall annular configuration.

For $U_s = 0$, Eqs. (22) and (39) recover the local non-electroosmotic annular formulation associated with [24]. For $We = U_s = 0$, Eq. (39) gives $v_3 = -1 + \left(\frac{K}{4}\right) \left[\frac{r^2 - \epsilon^2 - (r_2^2 - \epsilon^2) \ln\left(\frac{r}{\epsilon}\right)}{H} \right]$, with $H = \ln\left(\frac{r_2}{\epsilon}\right)$. At $\epsilon = 0.15$, $r_2 = 1.10$, and $K = -10$, its maximum is 0.236035 and the maximum difference from an independent numerical solution is 2.02×10^{-12} . A rigid-wall limit additionally sets $\phi = 0$ and specifies pressure gradient or flux.

4.5. Numerical accuracy check

A local benchmark of Eq. (28) uses $\epsilon = 0.15$, $r_2 = 1.10$, $K = -10$, $m_e = 2$, $U_s = 0.20$, and $\zeta_1 = \zeta_2 = 1$. With $y = v'_3$, the independent boundary-value system is $v'_3 = y$ and $y' = \frac{K - U_s m_e^2 \Phi - \frac{y}{r} - We \frac{y^2}{r}}{1 + 2We y}$, with $v_3(\epsilon) = v_3(r_2) = -1$. Adaptive collocation uses tolerance 10^{-9} and 201 initial nodes; comparison uses 4001 radial points. Table 1 compares the zeroth-, first-, and second-order profiles with this nonlinear solution. E_j is the maximum absolute velocity error through order j , and R_{∞} is the maximum momentum residual of the second-order profile.

Table 1. Local accuracy of the perturbation approximation $|K| = 10$.

We	χ	E_0	E_1	E_2	$R_{\infty}/ K $
0.0000	0.0000	2.211e-12	2.211e-12	2.211e-12	2.309e-15
0.0025	0.0240	5.596e-03	1.905e-04	6.633e-06	1.183e-03
0.0050	0.0473	1.084e-02	7.394e-04	5.075e-05	9.306e-03
0.0100	0.0924	2.036e-02	2.799e-03	3.735e-04	7.213e-02
0.0200	0.1797	4.287e-02	1.020e-02	2.573e-03	5.474e-01
0.0400	0.3641	1.054e-01	3.600e-02	1.591e-02	4.140e+00

Refinement from 2001 to 4001 comparison points changes E_2 at $We = 0.01$ by less than 10^{-12} . Independent integrated-stress quadrature agrees with the collocation solution within 2.30×10^{-12} on the finer grid. The E_2 ratios 7.65 and 7.36 on successive doubling of small We are consistent with the $O(We^3)$ remainder in Eq. (50). The larger error and residual at $We = 0.04$ delimit the accuracy of this benchmark; the table does not establish convergence for arbitrary forcing or We .

5. Results and Discussion

This section discusses the effects of electroosmotic forcing, Williamson rheology, wave amplitude, annular geometry, and wall flexibility on the axial velocity, shear stress, and streamline pattern. The main parameters are the Weissenberg number We , wave-amplitude ratio ϕ , electroosmotic parameter m_e , Helmholtz–Smoluchowski velocity U_s , wall zeta potentials ζ_1 and ζ_2 , radius ratio ϵ , and wall parameters $L_1 - L_5$. The parameter values are reported in the corresponding figure captions.

5.1. Axial velocity

Figures 2 - 4 show the axial velocity distribution in the annular gap. The profiles satisfy the no-slip condition at both boundaries and exhibit their largest values in the core region. This behaviour is consistent with pressure-driven and peristaltic transport in confined annular geometries.

Figure 2 illustrates the influence of the Weissenberg number and the wave-amplitude ratio. Increasing We enhances the axial velocity in the core region because the nonlinear Williamson contribution strengthens the shear-thinning response and reduces the effective flow resistance. Increasing ϕ also raises the velocity, since a larger wave amplitude intensifies the peristaltic pumping action and transfers more momentum to the fluid.

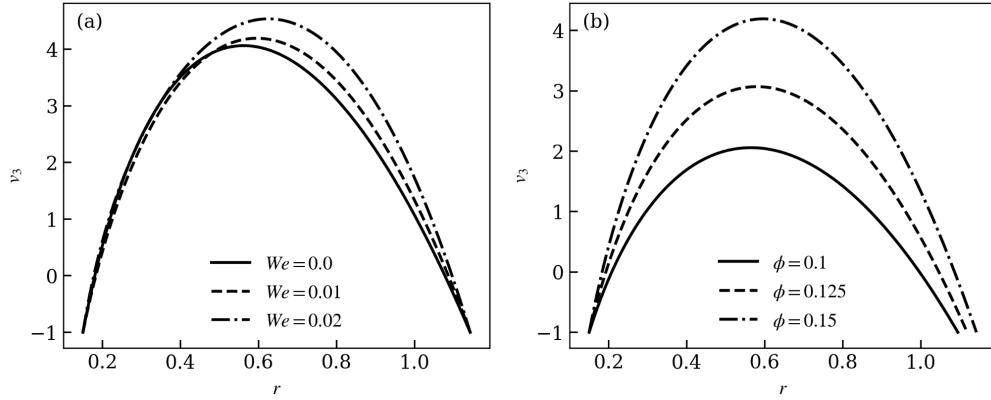


Figure 2. Axial velocity distribution $v_3(r)$ for different values of (a) the Weissenberg number We and (b) the amplitude ratio ϕ . The fixed parameters are $\epsilon = 0.15$, $L_1 = 0.1$, $L_2 = 0.5$, $L_3 = 0.1$, $L_4 = 0.1$, $L_5 = 0.1$, $t = 0.1$, $z = 0.4$, $m_e = 2.0$, $U_s = -0.20$, and $\zeta_1 = \zeta_2 = 1.0$.

Figure 3 presents the effects of the electroosmotic parameter and the outer-wall zeta potential. For the selected direction of U_s , increasing m_e strengthens the electroosmotic contribution in the axial momentum equation and shifts the velocity profile upward. Similarly, increasing ζ_2 increases the electric-potential contribution near the flexible wall and enhances electrokinetic driving. These results show that zeta potential can serve as an effective control parameter for electroosmotic peristaltic transport.

Figure 4 displays the effect of wall parameters. Variations in $L_1 - L_5$ alter the wall-induced pressure-gradient term K and therefore change the axial velocity. The flexural-rigidity, longitudinal-tension, inertia, damping, and spring-stiffness parameters modify the mechanical response of the flexible wall and influence the fluid motion through the coupled wall-pressure relation.

5.2. Shear stress

Figures 5 – 7 show the radial distribution of shear stress. The shear stress is positive near the inner wall and negative near the outer wall because the axial velocity reaches a maximum inside the annular gap and its radial derivative changes sign across the channel.

Figure 5 shows that increasing m_e increases the magnitude of the shear stress. Stronger electroosmotic forcing steepens the velocity gradient, which increases the positive stress near the

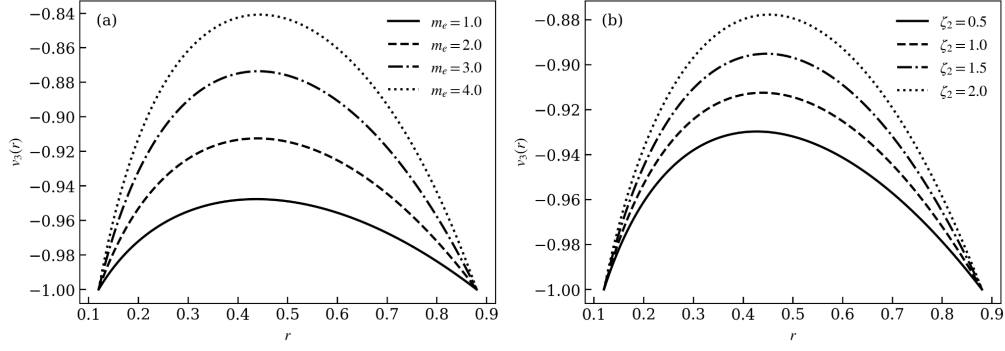


Figure 3. Axial velocity profile $v_3(r)$ for different values of (a) the electroosmotic parameter m_e at $\zeta_2 = 1.0$ and (b) the outer-wall zeta potential ζ_2 at $m_e = 2.0$. The remaining parameters are $\epsilon = 0.12$, $\phi = -0.12$, $We = 0.01$, $z = 0.25$, $t = 0$, $U_s = 0.20$, $\zeta_1 = 1.0$, and $(L_1, L_2, L_3, L_4, L_5) = (0.1, 0.5, 0.1, 0.1, 0.1)$.

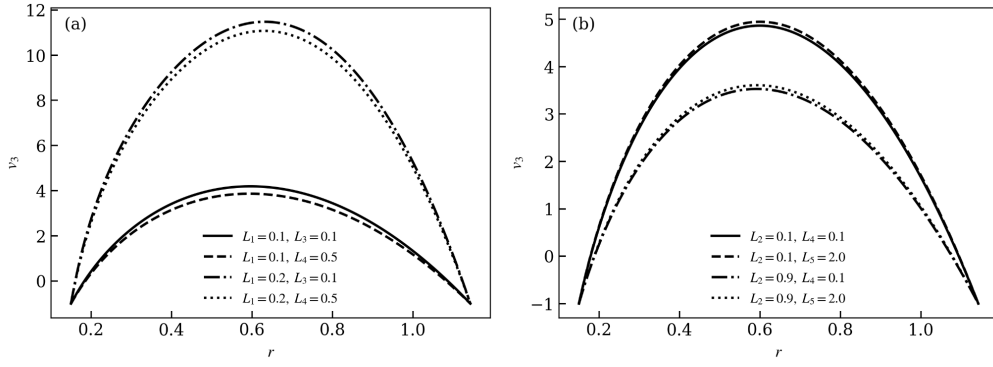


Figure 4. Axial velocity profile $v_3(r)$ for different wall-parameter combinations. Panel (a) shows the effects of L_1 , L_3 , and L_4 ; panel (b) shows the effects of L_2 , L_4 , and L_5 . The remaining parameters are $\epsilon = 0.15$, $\phi = 0.15$, $We = 0.01$, $z = 0.4$, $t = 0.1$, $m_e = 2.0$, $U_s = -0.20$, and $\zeta_1 = \zeta_2 = 1.0$.

inner wall and the negative stress near the outer wall. Increasing We also increases the stress magnitude through the nonlinear term $We(v'_3)^2$ in Eq. (51).

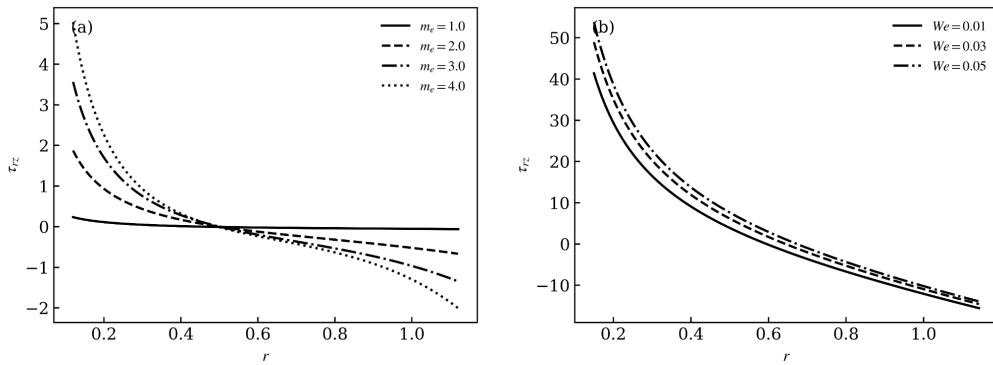


Figure 5. Shear-stress distribution τ_{rz} for different values of (a) the electroosmotic parameter m_e at fixed $We = 0.01$ and (b) the Weissenberg number We at fixed $m_e = 2.0$.

Figure 6 demonstrates that the shear-stress magnitude increases with both ϕ and ϵ . A larger wave amplitude strengthens the peristaltic mechanism and increases radial velocity gradients. A

larger radius ratio narrows the annular gap, thereby increasing the velocity gradient required to accommodate the same transport mechanism.

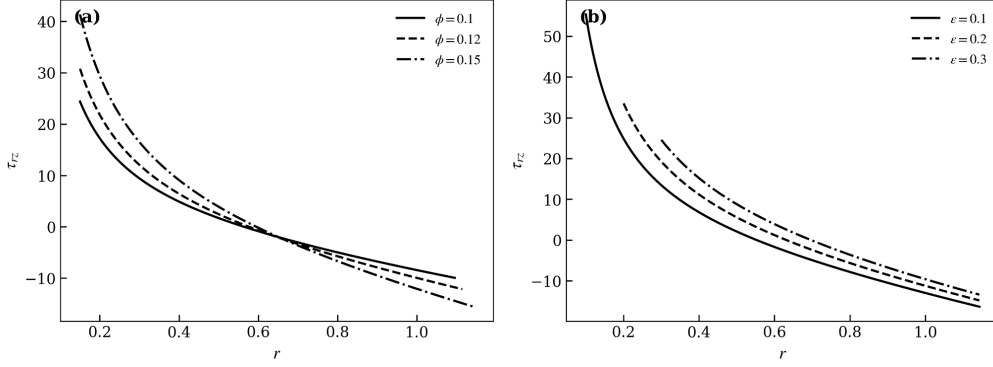


Figure 6. Shear-stress distribution τ_{rz} for different values of (a) the wave-amplitude ratio ϕ and (b) the radius ratio ϵ . The remaining parameters are $We = 0.01$, $m_e = 2.0$, $U_s = -0.20$, $\zeta_1 = \zeta_2 = 1.0$, $z = 0.4$, and $t = 0.1$.

Figure 7 confirms that the wall parameters have a measurable effect on the shear-stress distribution. Changes in flexural rigidity, longitudinal tension, wall inertia, damping, and spring stiffness alter the pressure-gradient term and consequently modify the boundary shear stress. This behaviour is relevant in physiological and microfluidic applications where wall shear stress must be controlled.

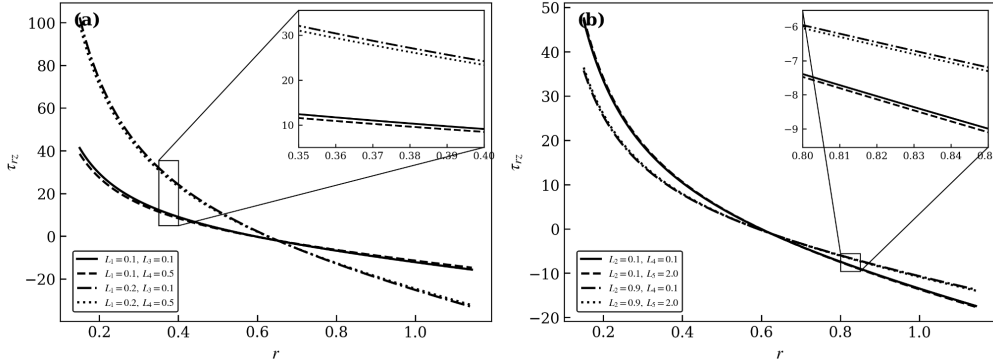


Figure 7. Shear-stress distribution τ_{rz} for different wall-parameter combinations. Panel (a) shows the effects of L_1 , L_3 , and L_4 ; panel (b) shows the effects of L_2 , L_4 , and L_5 . Insets show magnified regions of the curves. The remaining parameters are $\epsilon = 0.15$, $\phi = 0.15$, $We = 0.01$, $m_e = 2.0$, $U_s = -0.20$, $\zeta_1 = \zeta_2 = 1.0$, $z = 0.4$, and $t = 0.1$.

5.2.1. Compact quantitative comparison

Table 2 varies one input from the Section 4.5 baseline at $We = 0.01$. All other inputs remain fixed. The reported values use the second-order solution, and $q = \psi(r_2)$ is the local wave-frame flux integral in Eq. (57). Wall coefficients and wave amplitude enter through K and r_2 ; these local comparisons supplement Figures 2–7 without reconstructing their original settings. Parameter trends refer to the stated driving condition and signed stresses.

5.3. Streamlines and trapping

Figures 8 and 9 present the wave-frame streamlines. Closed streamlines indicate the formation of trapped boluses that move with the peristaltic wave. Figure 8 shows that electroosmotic forcing changes the size and shape of the trapped bolus by modifying the axial velocity distribution

Table 2. Local parameter comparison and second-order velocity error.

Changed input	$\max v_3$	τ_{rz} inner	τ_{rz} outer	q	E_2
Baseline	0.309409	10.075965	-4.356477	-0.089364	3.73e-04
$U_s = -0.2$	0.168734	8.931032	-3.847114	-0.144975	2.38e-04
$m_e = 4$	0.386559	10.820216	-4.778793	-0.055794	4.86e-04
$\zeta_2 = 2$	0.358511	10.392515	-4.572935	-0.068922	4.15e-04
$\epsilon = 0.2$	0.152595	8.286897	-4.147554	-0.144645	2.04e-04
$r_2 = 1.2$	0.605435	11.662127	-4.795905	0.028480	6.70e-04
$K = -15$	0.933692	15.078079	-6.373234	0.153283	1.63e-03

and stream-function contours. Figure 9 indicates that increasing We strengthens the closed-streamline pattern, reflecting the effect of Williamson rheology on bolus formation and recirculation, the Figures also provide a qualitative comparison of stream-function contours. Physical trapping is interpreted subject to impermeable-wall kinematics and constant wave-frame flux $q(z) = \psi(r_2(z), z)$: $v_1(\epsilon) = 0$, $v_1(r_2, z) = -\frac{dr_2}{dz}$, and $\frac{dq}{dz} = 0$. Bolus-volume measurements are outside the reported contour analysis. The trapping statements are subject to these compatibility conditions.

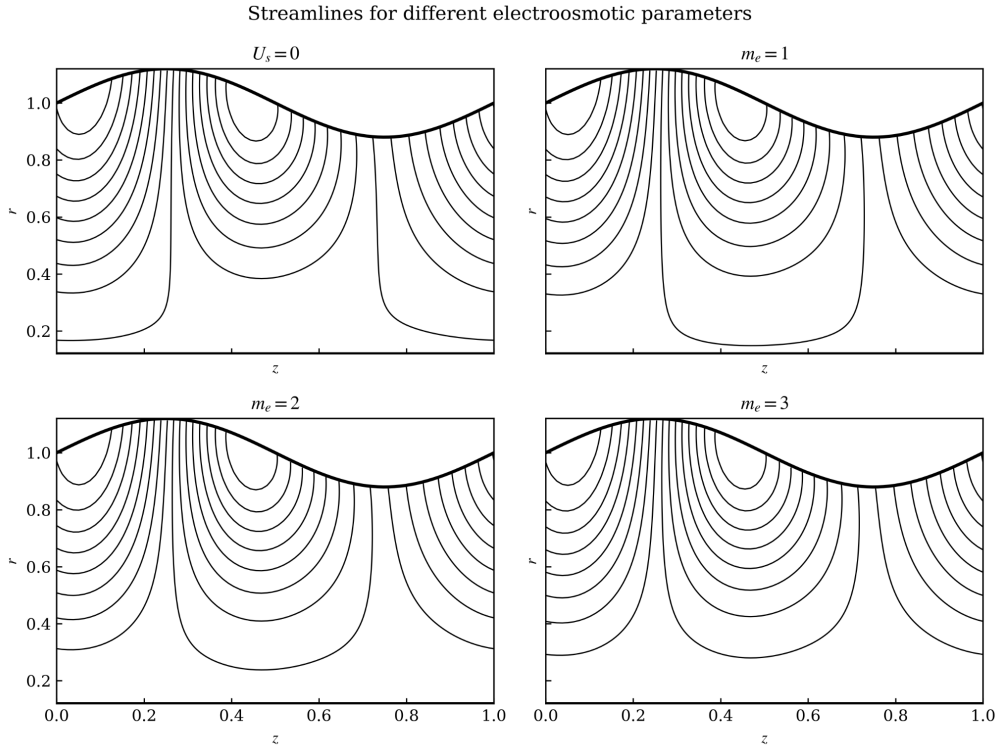


Figure 8. Wave-frame streamlines for different values of the electroosmotic parameter m_e . The solid curves represent contours of $\psi(r, z)$, while the lower and upper boundaries denote the inner rigid wall and flexible outer wall, respectively.

Overall, the results indicate that the axial velocity increases with We , ϕ , m_e , and ζ_2 for the selected parameter values. The shear-stress magnitude increases under stronger electroosmotic forcing, larger Weissenberg number, larger wave amplitude, and larger radius ratio. The streamline patterns demonstrate that both electroosmosis and Williamson rheology can be used to control trapping in a flexible annular channel.

A direct engineering analogue is a charged coaxial microchannel with a rigid inner insert and an actuated compliant outer sleeve. Catheter or endoscope configurations provide geometric

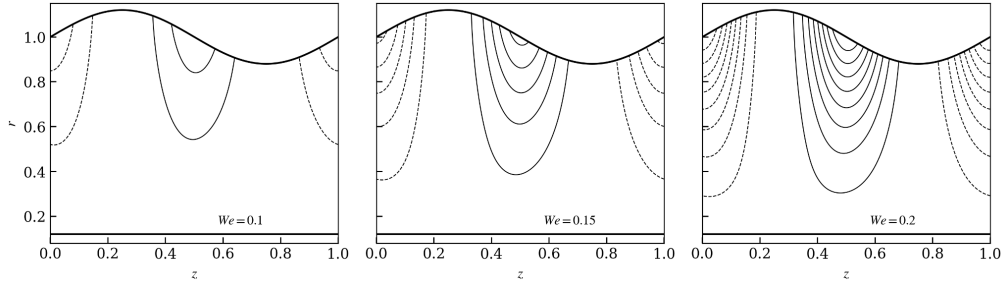


Figure 9. Wave-frame streamlines for different values of the Weissenberg number We . The fixed parameters are $\epsilon = 0.12$, $\phi = 0.12$, $L_1 = 0.1$, $L_2 = 0.5$, $L_3 = 0.1$, $L_4 = 0.1$, $L_5 = 0.1$, $t = 0$, $m_e = 2.0$, $U_s = -0.20$, and $\zeta_1 = \zeta_2 = 1.0$.

analogues under the stated approximation and rheological conditions. The results describe an idealized analytical system and parameter-dependent responses, rather than a calibrated clinical prediction.

6. Conclusions

An analytical formulation has been developed for electroosmotic peristaltic transport of a Williamson fluid through a flexible annular channel. The main conclusions are as follows:

- (i) The electroosmotic body force modifies the axial momentum equation through the electric-potential distribution governed by the Debye–Huckel approximation.
- (ii) The zeroth-order axial velocity contains a direct electroosmotic contribution, while the first- and second-order Williamson corrections are affected through the modified velocity gradients.
- (iii) Increasing the electroosmotic parameter and wall zeta potential enhances the axial velocity for the selected direction of the Helmholtz–Smoluchowski velocity.
- (iv) The Weissenberg number increases both the axial velocity and the shear-stress magnitude, confirming the importance of Williamson rheology.
- (v) Wall elasticity parameters influence the pressure gradient, velocity distribution, shear stress, and trapping structure.

The present formulation provides a compact framework for analysing electrokinetically controlled peristaltic transport in annular microfluidic and physiological systems.

Acknowledgments

The authors acknowledge the academic support of their institutions during the preparation of this work.

Funding

The authors received no financial support for the research, authorship, or publication of this work.

Data availability

Data sharing is not applicable to this article because the work is analytical and no datasets were generated or analyzed.

Conflict of interest

There is no conflict of interest to disclose.

Author contributions

All authors contributed equally and approved the final manuscript.

Declaration of using AI tools

Generative artificial intelligence was used solely to assist with manuscript formatting and language presentation. The authors reviewed and approved the final manuscript and take full responsibility for its content.

References

- [1] H. Hamid, T. Javed, B. Ahmed, and N. Ali, "Numerical study of two-dimensional non-Newtonian peristaltic flow for long wavelength and moderate Reynolds number," *J. Braz. Soc. Mech. Sci. Eng.*, vol. 39, no. 11, pp. 4421-4430, 2017. doi: 10.1007/s40430-017-0810-0
- [2] A. Kavitha, R. H. Reddy, R. Saravana, and S. Serenade, "Peristaltic transport of a Jeffrey fluid in contact with a Newtonian fluid in an inclined channel," *Ain Shams Eng. J.*, vol. 8, no. 4, pp. 683-687, 2017. doi: 10.1016/j.asej.2015.10.014
- [3] M. Gudekote and R. Choudhari, "Slip effects on peristaltic transport of Casson fluid in an inclined elastic tube with porous walls," *J. Adv. Res. Fluid Mech. Therm. Sci.*, vol. 43, no. 1, pp. 67-80, 2018.
- [4] C. Rajashekhar, G. Manjunatha, K. V. Prasad, B. B. Divya, and H. Vaidya, "Peristaltic transport of two-layered blood flow using Herschel-Bulkley model," *Cogent Eng.*, vol. 5, no. 1, Art. no. 1495592, 2018. doi: 10.1080/23311916.2018.1495592
- [5] T. Javed, B. Ahmed, A. H. Hamid, and M. Sajid, "Numerical analysis of peristaltic transport of Casson fluid for non-zero Reynolds number in presence of the magnetic field," *Nonlinear Eng.*, vol. 7, no. 3, pp. 183-193, 2018. doi: 10.1515/nleng-2017-0098
- [6] T. W. Latham, "Fluid motions in a peristaltic pump," Ph.D. dissertation, Massachusetts Institute of Technology, Cambridge, MA, USA, 1966.
- [7] A. H. Shapiro, M. Y. Jaffrin, and S. L. Weinberg, "Peristaltic pumping with long wavelengths at low Reynolds number," *J. Fluid Mech.*, vol. 37, no. 4, pp. 799-825, 1969. doi: 10.1017/S0022112069000899
- [8] F. C. P. Yin and Y. C. Fung, "Comparison of theory and experiment in peristaltic transport," *J. Fluid Mech.*, vol. 47, no. 1, pp. 93-112, 1971. doi: 10.1017/S0022112071000958
- [9] G. Radha Krishnamacharya and Ch. Srinivasulu, "Influence of wall properties on peristaltic transport with heat transfer," *C. R. Mecanique*, vol. 335, pp. 369-373, 2007. doi: 10.1016/j.crme.2007.05.002
- [10] S. Srinivas and M. Kothandapani, "The influence of heat and mass transfer on MHD peristaltic flow through a porous space with compliant walls," *Appl. Math. Comput.*, vol. 213, pp. 197-208, 2009. doi: 10.1016/j.amc.2009.02.054
- [11] Kh. S. Mekheimer, Y. Abd elmaboud, and A. I. Abdellateef, "Peristaltic transport through eccentric cylinders: Mathematical model," *Appl. Bionics Biomech.*, vol. 10, pp. 19-27, 2013. doi: 10.1155/2013/902097
- [12] T. Hayat, S. Bibi, M. Rafiq, A. Alsaedi, and F. M. Abbasi, "Effect of an inclined magnetic field on peristaltic flow of Williamson fluid in an inclined channel with convective conditions," *J. Magn. Magn. Mater.*, 2015. doi: 10.1016/j.jmmm.2015.10.107
- [13] H. Yasmin, T. Hayat, N. Alotaibi, and H. Gao, "Convective heat and mass transfer analysis on peristaltic flow of Williamson fluid with Hall effects and Joule heating," *Int. J. Biomath.*, vol. 7, 2014. doi: 10.1142/S1793524514500582
- [14] R. Ellahi, A. Riaz, and S. Nadeem, "Three-dimensional peristaltic flow of Williamson fluid in a rectangular duct," *Indian J. Phys.*, vol. 87, pp. 1275-1281, 2013. doi: 10.1007/s12648-013-0340-2
- [15] M. M. Bhatti, M. Sheikholeslami, and A. Zeeshan, "Entropy analysis on electro-kinetically modulated peristaltic propulsion of magnetized nanofluid flow through a microchannel," *Entropy*, vol. 19, Art. no. 481, 2017. doi: 10.3390/e19090481
- [16] J. Prakash and D. Tripathi, "Electroosmotic flow of Williamson ionic nano liquids in a tapered microfluidic channel in presence of thermal radiation and peristalsis," *J. Mol. Liq.*, vol. 256, pp. 352-371, 2018. doi: 10.1016/j.molliq.2018.02.043

- [17] D. Tripathi, A. Yadav, O. A. Beg, and R. Kumar, "Study of microvascular non-Newtonian blood flow modulated by electroosmosis," *Microvasc. Res.*, vol. 117, pp. 28-36, 2018. doi: 10.1016/j.mvr.2018.01.001
- [18] S. Noreen, S. Waheed, A. Hussanan, and D. Lu, "Analytical solution for heat transfer in electroosmotic flow of a Carreau fluid in a wavy microchannel," *Appl. Sci.*, vol. 9, no. 20, Art. no. 4359, 2019. doi: 10.3390/app9204359
- [19] M. Nazeer, F. Hussain, M. Turkyilmazoglu, and Z. Ali, "Towards an approximate solution of highly viscous electro-osmotic flows in inclined channel: Applications in petroleum and gas engineering," *J. Magn. Magn. Mater.*, vol. 577, Art. no. 170793, 2023. doi: 10.1016/j.jmmm.2023.170793
- [20] A. Imran, M. A. Z. Raja, M. Shoaib, M. Zeb, and K. S. Nisar, "Electro-osmotic transport of a Williamson fluid within a ciliated microchannel with heat transfer analysis," *Case Stud. Therm. Eng.*, vol. 45, Art. no. 102904, 2023. doi: 10.1016/j.csite.2023.102904
- [21] S. Almutairi, F. Hussain, M. Nazeer, S. Saleem, and R. S. Mohammed, "Perturbation solution of multiphase flow of Williamson fluid through convergent and divergent conduits: Electro-osmotic effects," *Mod. Phys. Lett. B*, Art. no. 2450348, 2024. doi: 10.1142/S0217984924503482
- [22] F. Hussain, M. Nazeer, M. Jameel, N. Nazarova, R. Ghodhbani, L. Abu El Maati, and M. I. Khan, "Analytical study of thermal transport in microchannels with hafnium nanoparticles suspended in Williamson fluid under electro-kinetic force," *Case Stud. Therm. Eng.*, vol. 72, Art. no. 106383, 2025. doi: 10.1016/j.csite.2025.106383
- [23] S. O. Salawu, H. A. Ogunseye, E. O. Fatunmbi, and M. D. Shamshuddin, "Energy optimization in zeta potential electroosmotic Casson-Williamson viscous heating magnetized fluid in a convective cooling microchannel," *Appl. Therm. Eng.*, vol. 280, Art. no. 128382, 2025. doi: 10.1016/j.applthermaleng.2025.128382
- [24] D. G. S. Al-Khafajy and W. N. Al-Delfi, "The peristaltic flow of Williamson fluid through a flexible channel," *Iraqi J. Sci.*, vol. 64, no. 2, pp. 865-877, 2023. doi: 10.24996/ijss.2023.64.2.32



All open access articles published in Transactions on Computational Modeling and Intelligent Systems (<http://tcmis.org>) are distributed under the terms of the CC BY-NC 4.0 license (Creative Commons Attribution Non-Commercial 4.0 International Public License as currently displayed at <http://creativecommons.org/licenses/by-nc/4.0/legalcode>) which permits unrestricted use, distribution, and reproduction in any medium, for non-commercial purposes, provided the original work is properly cited.