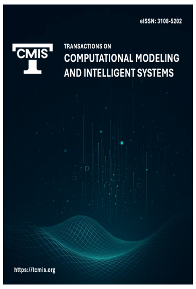

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RESEARCH ARTICLE

Simulation analysis of multi-objective one-dimensional cutting problems

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Abstract

In this paper, we compare published studies about optimization models for multi-objective one-dimensional cutting problems. Although all of the models address multiple objectives, they implement these goals in different ways. Some focus primarily on minimizing the amount of waste while setting limits on reusable bars returned to stock, whereas others combine these criteria with additional factors, such as the number of affected bars and the number of cuts. First, we provide a brief overview of these mathematical models, including one developed and published by the authors earlier, as well as three others from the existing literature that pursue objectives very similar to ours. In addition to comparing the models based on the included objectives such as the number of cuts, the amount of waste, and the number of reusable bars, our goal was also to analyse the computational requirements and limitations of the models in case of large-scale problems.

Keywords: Mixed integer programming, one-dimensional cutting problems, simulation

1. Introduction

The one-dimensional cutting stock problem (1DCSP) involves a set of available one-dimensional stock items (referred to as bars) in order to serve smaller required pieces (referred as orders) in specified lengths and quantities.

In today's competitive industrial environment the optimization problems related to cutting raw materials are of critical importance for companies. Cutting stock problems have numerous application areas in industrial sectors, such as construction and civil engineering [1], steel industry [2], woodworking [3] and many other fields, where efficient solutions have a direct impact on production costs and the amount of waste generated. More recently, [4] combined building information modelling (BIM) with a multi-objective cutting-pattern optimization to reduce rebar cutting waste in construction projects, again balancing several practical objectives within a single pipeline. Researchers have moved beyond the classical single-objective CSP model, which focuses solely on waste minimization, and now focus on more complex multi-objective variants that more accurately reflect real industrial environments. In practice, multi-objective one-dimensional cutting optimization models may aim to achieve several objectives simultaneously, including:

- minimizing the number of required cuts,

- reducing the amount of waste produced,
- minimizing the number of stock bars involved.

The following research directions have received particular attention:

- **Integrated models:** Several studies, such as [5], focus on integrating the lot-sizing and cutting stock problems. These kinds of studies investigate how production quantities and cutting patterns can be optimized simultaneously, with the aim of minimizing setup costs and inventory levels.
- **Side constraints:** There is a growing number of models that incorporate additional practical considerations. Model of [6] implements cutting setups and open stacks, which are critical in the paper and tube manufacturing industries for improving production efficiency and reducing logistical challenges.
- **Usable leftovers**: The handling of reusable remainders, which is also a central concern of MACROPT-1D, has recently been revisited in the literature. Senergues et al. [7] published an updated and comprehensive review of the cutting stock problem with usable leftovers (CSPUL), extending the earlier survey of Cherri et al. [8] with more than fifty studies published after 2008; unlike our comparative simulation study, however, this work remains a literature synthesis and does not provide a numerical, model-to-model comparison.

  Ma et al. [9] extended the usable-leftover concept to a two-dimensional, multi-objective steel-cutting problem minimizing waste area, order overrun, and slitter adjustments. As the problem is two-dimensional and thus structurally different from the models compared here, it is solved with an iterated local search heuristic rather than an exact MILP solver.
- **New algorithmic approaches:** Due to the computational complexity of the CSP, new heuristics and metaheuristics are continuously being developed. Recent studies also demonstrate the increasing adoption of artificial intelligence–based methods, such as [10] and tree-based heuristics [11], which aim to significantly reduce computational time while maintaining solution quality. Similarly, [12] proposed a column-generation-based local branching matheuristic for a multi-period cutting stock problem with due dates and setups, in which the objective combines tardiness, earliness, and setup costs; their results confirm that as additional objectives and side constraints are introduced, exact solution approaches—such as the four MILP models compared in this paper—face rapidly increasing computational demands, which is consistent with the runtime behavior we report in Section 3.6.

1.1. Contributions of this study

The aim of this study is to analyze and compare our previously published **M**ultiobjective, **A**djustable **C**ost and **R**ecycling parametric **OPT**imization model for **1D**imensional cutting stock problems (MACROPT-1D) [13] with three other theoretical models proposed in the literature: the two mixed-integer linear programming formulations of Abuabara and Morabito (A&Mv1 and A&Mv2) [14], and the multi-objective model of Trkman and Gradišar (T&G) [15]. The comparison considers not only the objectives of the models, such as cost or waste reduction, but also implementation aspects, practical applicability, and computational performance, including runtime behavior. Although several multi-objective formulations of the one-dimensional cutting stock problem exist, including the models discussed above, they are typically validated either on small illustrative examples or evaluated only with respect to their own native objective function. As a result, it remains unclear how these models behave on criteria that are not part of their own objective, for instance, the number of cuts produced by a waste-only model, or the number of stock bars affected by a cost-minimizing model. This gap motivates the specific contributions of the present study, which can be summarized as follows.

- We conduct a large-scale simulation study comprising 900 randomly generated instances, structured into nine scenario groups along two independent dimensions (stock level and order volume), in contrast to the small numerical examples typically used to validate such models in the literature.

- We cross-evaluate all four models with respect to five criteria simultaneously: the number of cuts, the amount of waste, the number of affected bars, the total cost, and the runtime, regardless of which of these criteria is part of a model's own objective, in order to reveal trade-offs that may remain hidden when a model is assessed only against its native objective function.
- We investigate whether the parametric flexibility of MACROPT-1D, which allows the relative importance of cutting cost, waste, and the number of bars used to be tuned through three independent coefficients, without altering the model structure, translates into a measurable advantage over models with a fixed or more restrictive objective, such as A&Mv1, A&Mv2, and T&G.
- We examine the theoretical correctness of the compared formulations, with particular attention to the logical relationship between the binary indicator variables governing bar usage and the return of remainders to stock in the A&Mv2 model.

2. Theoretical background of selected 1DCS models

In this section we provide a brief overview of the models that were implemented, analyzed, and compared in Python. The models presented in this study are based on the papers reviewed in the survey [8], complemented by our MACROPT-1D model published in [13]. In [8], the authors highlight and discuss the following one-dimensional cutting stock models: two versions of a waste minimization model proposed by Abuabara and Morabito [14] and the multi-objective model of Trkman and Gradišar [15], which incorporates not only the amount of waste but also the cost associated with returning reusable-length bars to inventory within the objective function. These models were selected for comparison and analysis. In the following section, we provide an overview of the models, beginning with the general notation in table 1.

Table 1. Notation.

Input data	
Symbol	Description
$B_1, B_2, \ldots, B_m$	Bars available in stock.
$O_1, O_2, \ldots, O_n$	Orders, that should be served.
$bl_1, bl_2, \ldots, bl_m$	Length of the bars in stock.
$ol_1, ol_2, \ldots, ol_n$	Requested length of the orders.
W	Waste limit, below which a remaining part is considered waste.
cC	Cost of a cut.
wP	Penalty associated with the generated waste, expressed on a per-unit basis.
rC	Recycling cost, i.e. the cost of returning a part back to stock.
Variables, technological constants	
Symbol	Description
x_{ij}	Elements of an $n \times m$ binary matrix assigning orders to stock bars.
$Rl_1, Rl_2, \ldots, Rl_m$	Residual length of a stock bar after cutting.
$Wl_1, Wl_2, \ldots, Wl_m$	Waste length of a stock bar after cutting.
$YU_1, YU_2, \ldots, YU_m$	Binary variable indicating whether a bar is involved.
$YR_1, YR_2, \ldots, YR_m$	Binary variable indicating whether a stock bar has a remainder.
$YRC_1, YRC_2, \ldots, YRC_m$	Binary variable indicating whether the remainder of a stock bar is reusable.
$YRT_1, YRT_2, \ldots, YRT_m$	Binary variable indicating whether the remainder returns to stock.
M	Large enough constant.
ϵ	Small enough constant.

2.1. MACROPT-1D

We developed a multi-objective mixed-integer linear programming (MILP) model for one-dimensional cutting stock problem. The model minimizes a linear combination of the number of cuts, weighted by the cut cost coefficient (cC), the resulting waste, weighted by the waste coefficient (wP) and the number of stock bars used to fulfill the orders. The model includes constraints that set remainder lengths (Rl_j) and the binary indicator variables (Wl_j, YU_j) according to the waste length limit and utilization of stock bars $j = 1, 2, \ldots, m$. In this section we give a brief overview of the model, for detailed explanation see [13].

$$\begin{aligned} \min \quad & cC \cdot \sum_{j=1}^{m} \Big(\sum_{i=1}^{n} (x_{ij}) - 1 + YR_j \Big) \\ & + wP \cdot \sum_{j=1}^{m} Wl_j \\ & + \sum_{j=1}^{m} YU_j \end{aligned} \tag{1}$$

st.

$$\sum_{i=1}^{n} x_{ij} ol_i + Rl_j = bl_j \qquad j = 1, 2, \ldots, m \tag{2}$$

$$\sum_{j=1}^{m} x_{ij} = 1 \qquad i = 1, 2, \ldots, n \tag{3}$$

$$YR_j \geq \frac{Rlj}{bl_j} \qquad j = 1, 2, \ldots, m \tag{4}$$

$$Rl_j - Wl_j \leq M \cdot YRC_j \qquad j = 1, 2, \ldots, m \tag{5}$$

$$W - Rl_j \leq M \cdot (1 - YRC_j) \qquad j = 1, 2, \ldots, m \tag{6}$$

$$YU_j \geq 1 - \frac{Rl_j}{bl_j} \qquad j = 1, 2, \ldots, m. \tag{7}$$

Objective function (1) combines three goals: minimizing the cutting cost, the waste penalty and the number of stock bars used. Constraint (2) ensures that the length of each stock bar is equal to the total length of the orders assigned to it plus the length of the remainder. Equation (3) guarantees a unique assignment of every order to a stock bar. (4) sets the appropriate value of the binary variables YR_j, while the subsequent pair of constraints provides an if-then logical formulation that equates the waste length to the remainder length whenever it is below the waste limit. Finally, (7) sets the value of the binary variables YU_j depending on their usage.

2.2. 1st version model of Abuabara and Morabito

The next model is a mixed integer linear programming model from Abuabara and Morabito, [14] (A&Mv1). This single objective model was developed for solving the cutting stock problem arising in the processing of metal structural tubes with the minimization of the waste. We give a short overview of the model, for more detailed explanation see [14].

$$\min \sum_{j=1}^{m} Wl_j \tag{8}$$

$$\sum_{i=1}^{n} x_{ij} ol_i + Rl_j = bl_j \qquad j = 1, 2, \ldots, m \tag{9}$$

$$\sum_{j=1}^{m} x_{ij} = 1 \qquad i = 1, 2, \ldots, n \tag{10}$$

$$YU_j \leq \sum_{i=1}^{n} x_{ij} \qquad j = 1, 2, \ldots, m \tag{11}$$

$$M \cdot YU_j \geq \sum_{i=1}^{n} x_{ij} \qquad j = 1, 2, \ldots, m \tag{12}$$

$$Rl_j - W \geq -M \cdot (1 - YRC_j) + \epsilon \qquad j = 1, 2, \ldots, m \tag{13}$$

$$Rl_j - W \leq M \cdot YRC_j \qquad j = 1, 2, \ldots, m \tag{14}$$

$$Wl_j - M \cdot (1 - YRC_j) \leq 0 \qquad j = 1, 2, \ldots, m \tag{15}$$

$$Wl_j - M \cdot YU_j \leq 0 \qquad j = 1, 2, \ldots, m \tag{16}$$

$$-Rl_j + Wl_j \leq 0 \qquad j = 1, 2, \ldots, m \tag{17}$$

$$Rl_j - Wl_j + M \cdot (1 - YRC_j) + M \cdot YU_j \leq 2M \qquad j = 1, 2, \ldots, m \tag{18}$$

$$-YU_j + YRT_j \leq 0 \qquad j = 1, 2, \ldots, m \tag{19}$$

$$(1 - YRC_j) + YRT_j \leq 1 \qquad j = 1, 2, \ldots, m \tag{20}$$

$$YU_j - (1 - YRC_j) - YRT_j \leq 0 \qquad j = 1, 2, \ldots, m \tag{21}$$

$$\sum_{j=1}^{m} YRT_j \leq 1 \tag{22}$$

Objective (8) minimizes the total waste generated during the cutting process. Constraint (9), and (10) relate to the total length and order-to-bar assignments, analogous to (2) and (3). In Abuabara's 1st version model, the values of the binary variables YU_j, which represent bar usage, are determined by constraints (11) and (12), while the variables YRC_j are set by (13) and (14) using the waste limit W. Following constraints (15) and (16) ensure that if the remainder is reusable (i.e. $YRC_j = 1$) then no waste is generated on bar j (i.e. $Wl_j = 0$), and vice versa. This requirement is extended with (17), which guarantees that the length of the waste cannot be larger than the length of the remainder. Next one is a compound logical relationship that ensures the coordinated assignment of values to the corresponding variables. The constraint (18) restricts the values of the remainder and waste variables only if the given stock bar is used ($YU_j = 1$) and not reusable ($YRC_j = 0$). In this case, the entire remainder must be considered as waste. (19) states that a remainder can be returned to stock (YRT_j) if the corresponding bar has actually been used ($YU_j = 1$), while (20) ensures that a non reusable remainder ($YRC_j = 0$) cannot be returned to stock ($YRT_j = 0$). Finally, the last two constraints (21) and (22) formulate a special requirement of the real life application, they operate such that if multiple remainders of reusable length are generated, the model allows only one of them to be actually returned to stock. In all other cases, the variable YRC_j is forced to zero. This is achieved by modifying the cutting plan so that the corresponding remainder lengths do not reach the reusability threshold.

2.3. 2nd version model of Abuabara and Morabito

The authors of [14] simplified the previous model and corrected some infeasibility issues in the second version. We now provide a brief overview of this revised model (A&Mv2). Detailed explanation can be found in the mentioned paper.

$$\min \sum_{j=1}^{m} Wl_j \tag{23}$$

$$\sum_{i=1}^{n} x_{ij} ol_i \leq bl_j \qquad j = 1, 2, \ldots, m \tag{24}$$

$$\sum_{j=1}^{m} x_{ij} = 1 \qquad i = 1, 2, \ldots, n \tag{25}$$

$$W \cdot YRT_j \leq bl_j \cdot YU_j - \sum_{i=1}^{n} x_{ij} ol_i \qquad j = 1, 2, \ldots, m \tag{26}$$

$$Wl_j + YRT_j \cdot M \geq bl_j \cdot YU_j - \sum_{i=1}^{n} x_{ij} ol_i \qquad j = 1, 2, \ldots, m \tag{27}$$

$$\sum_{j=1}^{m} YRT_j \leq 1 \tag{28}$$

Besides the conditions identical to those of the first version, constraint (26) establishes a relationship between the remainder length on a given bar and its return to stock. Furthermore, constraint (27) ensures that any unused material is classified either as waste or as a sufficiently long remainder returned to inventory.

2.4. Multi-objective model of Trkman and Gradisar

Finally, we selected a model very similar to ours, which incorporates a linear combination of multiple goals in its objective [15]. In this section we give a brief overview of the Trkman model (T&G), for more details see the mentioned paper.

$$\min \sum_{j=1}^{m} Wl_j \cdot wP + YRT_j \cdot rC \tag{29}$$

$$\sum_{i=1}^{n} x_{ij} ol_i + Rl_j = bl_j \cdot YU_j \qquad j = 1, 2, \ldots, m \tag{30}$$

$$\sum_{j=1}^{m} x_{ij} = 1 \qquad i = 1, 2, \ldots, n \tag{31}$$

$$W - Rl_j + W \cdot (YRT_j - 1) \leq 0 \qquad j = 1, 2, \ldots, m \tag{32}$$

$$Rl_j - Wl_j - (YRT_j + (1 - YU_j)) \cdot \max_k \{bl_k\} \leq 0 \qquad j = 1, 2, \ldots, m \tag{33}$$

Constraints (30) and (31) are about the length balance and order-to-bar assignment analogous to those in the previous models. (32) ensures that the remainder is returned to stock if and only if its length exceeds the waste limit W. Constraint (33) manages the two possible states of a remainder, namely, being returned to stock or being treated entirely as waste.

3. Computational tests and comparison

3.1. Experimental Design and Computational Environment

The models were implemented in the Python programming language and solved by GUROBI solver. For the purposes of the simulation experiments, the test instances were categorized based on the level of the stock and volume of the orders into the groups presented in Table 2. For each scenario (LIL-LOV, ..., HIL-HOV), 100 randomly generated problem instances were created, each comprising the number and dimensions of stock bars as well as the orders.

Although fixed priority coefficients were employed for the different objectives (number of cuts, amount of waste, and number of stock bars used), the results were also analyzed separately according to each objective. The corresponding input parameters were as follows:

- Cost of cuts was fixed to 400, i.e. $cC = 400$.
- Penalty associated with the waste, on a per unit basis was fixed to 100, i.e. $wP = 100$.
- Recycling cost was 200, i.e. $rC = 200$.
- We used a waste limit fixed to 45, i.e. $W = 45$.

In the graphs, plots, and figures, the models are identified using the following colors and abbreviations:

Table 2. Scenario groups.

	Low Inventory Level (LIL) $5 < m \leq 80$	Medium Inventory Level (MIL) $80 < m \leq 100$	High Inventory Level (HIL) $100 < m \leq 150$
Low Order Volume (LOV) $5 < n \leq 10$	LIL - LOV	MIL - LOV	HIL - LOV
Medium Order Volume (MOV) $10 < n \leq 20$	LIL - MOV	MIL - MOV	HIL - MOV
High Order Volume (HOV) $20 < n \leq 30$	LIL - HOV	MIL - HOV	HIL - HOV

- Multi-objective, adjustable cost and recycling parametric optimization model [13] - MACROPT-1D, is highlighted by blue.
- First version of the mixed integer linear programming model of Abuabara and Morabito [14], (A&Mv1) is highlighted by orange.
- For the second version of the mixed integer linear programming model of Abuabara and Morabito [14], (A&Mv2) we use red.
- The multi-objective model of Trkman and Gradisar (T&G) is labelled by green.

Experiments were conducted on a Toshiba Satellite C50-A-1JV PC with Intel Pentium N3520 4 core CPU (2.16 GHz), and 4 GB DDR3 RAM.

3.2. Comparison of the number of cuts

In this section, we compare the number of cuts as a performance metric. Although the objective functions of the models differ, it is essential to evaluate the optimal solutions with respect to this common criterion, namely the number of cuts, irrespective of the associated cost coefficient. Figure 1 presents box plots of the number of cuts, grouped by stock level across three different scenarios (LIL, MIL, and HIL).

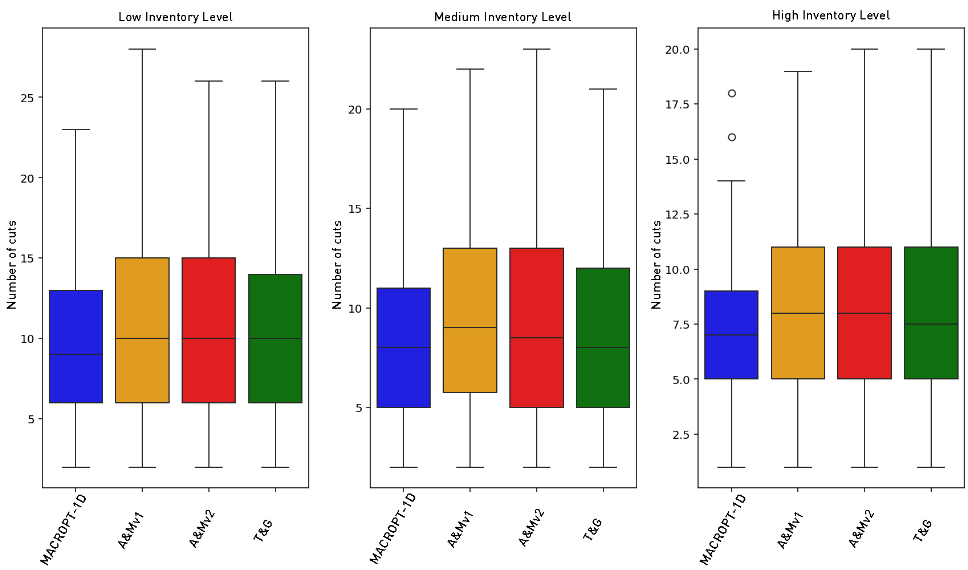

Figure 1. Comparison of the number of cuts in the optimal solutions across models

The objective functions of models A&Mv1 and A&Mv2 focus on waste minimization and do not explicitly account for the number of cuts; therefore, it is expected that these models perform the worst with respect to this criterion. In contrast, the MACROPT-1D and T&G models employ similar multi-objective formulations, in which identical cost coefficients were applied. The results shown in the figures indicate that MACROPT-1D offers better cutting plans in terms of the number of cuts, and consequently, in cutting costs as well.

3.3. Comparison of the generated waste

In this section, we focus exclusively on the amount of waste generated. This objective is common to all four models. The objective functions of the A&Mv1 and A&Mv2 models are solely devoted to waste minimization, whereas the MACROPT-1D and T&G models include waste minimization in combination with other objectives.

Figure 2 shows the boxplots of generated waste in the three mentioned category, from low to high stock level.

The first observation we highlight, is that cutting tasks generate less waste when a larger inventory is available. This outcome is logically expected and is consistently confirmed by the simulation results. This is unsurprising, as a greater stock variety provides increased flexibility in matching orders to stock lengths, thereby enabling more efficient material utilization.

A more unexpected result, however, is that although the A&Mv1 and A&Mv2 models exclusively aim at waste minimization, they nevertheless produce cutting plans associated with the highest amounts of waste compared to the other two models (MACROPT-1D and T&G). This behavior is presumably attributable to the constraint limiting the number of stock bars returned to the inventory. This restriction is intended to prevent the accumulation of a large variety of stock bars with different lengths, which may be difficult to track and could increase administrative complexity.

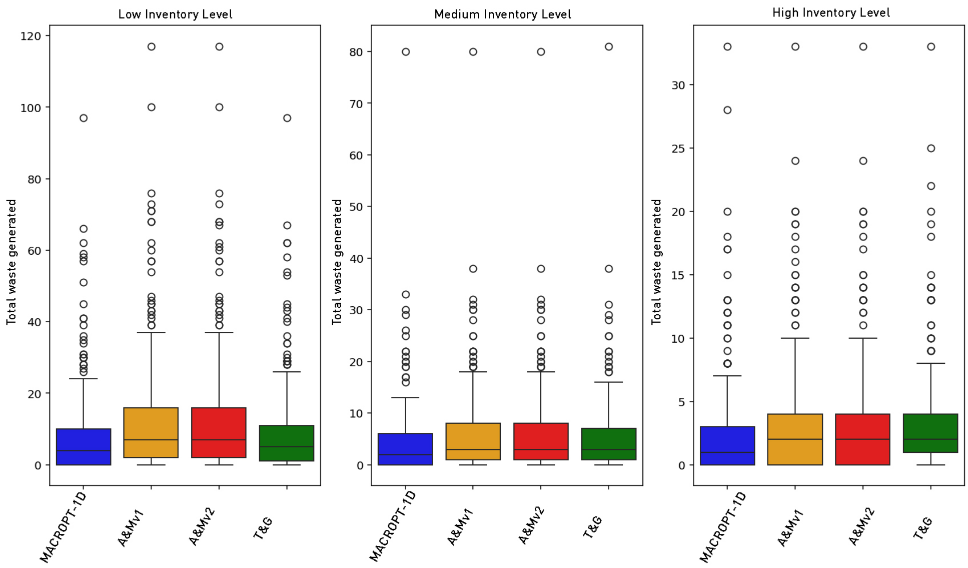

Figure 2. Comparison of the waste generated in the optimal solutions across models

3.4. Comparison of the affected bars

If the objective is waste minimization itself, the most straightforward approach would be to generate cutting patterns in which the residual length just exceeds the reusability threshold W. In this case, the leftover is not classified as waste but is returned to inventory. While this strategy is mathematically optimal, it is not necessarily desirable from a business perspective, as it may lead to inventory explosion and excessive heterogeneity.

For this reason, the A&Mv1 and A&Mv2 models attempt to restrict the generation of recycled stock by enforcing constraints (22) and (28), thereby limiting the number of reusable leftovers. From a managerial standpoint, this consideration can be highly relevant; however, it also entails the generation of a larger amount of residual material classified as waste, as discussed in the previous section.

In this section, we therefore examine the results from the perspective of inventory heterogeneity by comparing the models in terms of the number of stock bars affected (used) during the cutting process.

Figure 3 clearly shows that, in order to achieve exceptionally low levels of waste, the MACROPT-1D and T&G models require the involvement of a larger number of stock bars.

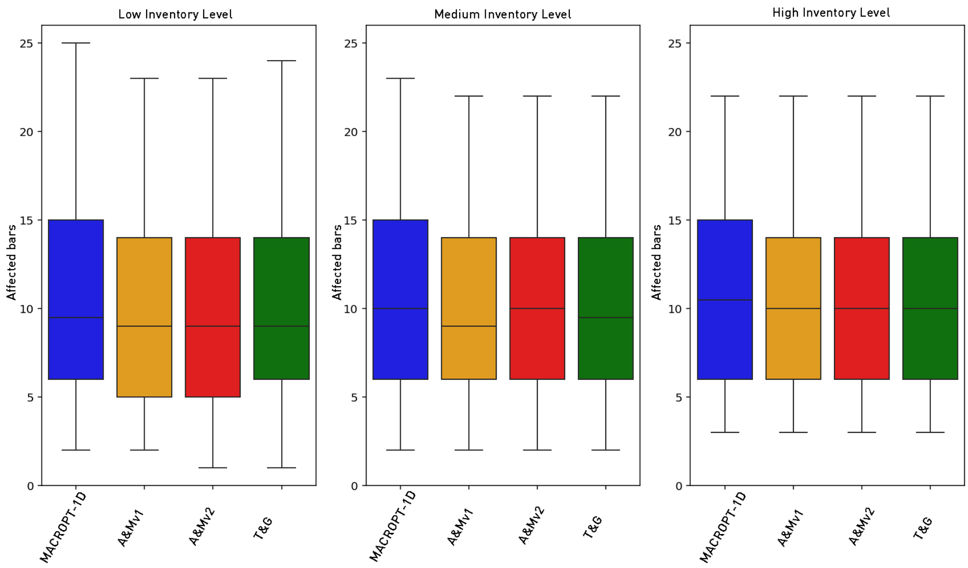

Figure 3. Comparison of the bars affected across models

3.5. Comparison of total cost

Using the cost coefficients introduced in Section 3.1, we compare the total cost of the cutting problems, which includes the cost of cuts, penalties for waste, and the impact on stock bars. Since each model focuses on a different objective, the total cost calculated using these general random cost coefficients may yield a different ranking. Therefore, this comparison is provided here solely to complete the analysis and should not be considered a definitive basis for model evaluation.

Figure 4 contains the boxplots of total cost in the three stock levels.

3.6. Comparison of runtime

An exponential function of the form $T(x) = a \cdot e^{bx}$ was fitted to the measured runtimes, and the resulting approximations are shown in Figure 5. For all models, an exponentially increasing trend is observed with respect to the number of orders. The three scenarios illustrate how inventory levels affect the rate of this increase. In general, it can be stated that beyond a certain critical number of orders, the runtimes of the models diverge, with A&Mv1 exhibiting the highest computational demand and the T&G model being the least resource-intensive.

4. Theoretical inaccuracy of the A&Mv2 model and a necessary correction

In this section we present a theoretical inaccuracy of the A&Mv2 model and propose a correction. Authors of [14] state that for the A&Mv2 model, YRT_j indicates if the slackness Rl_j of a used object j is a leftover, i.e., if $Rl_j \geq W (YRT_j = 1)$

$$YRT_j = \begin{cases} 1 & \text{if bar } j \text{ was used AND the leftover } Rl_j \geq W \\ 0 & \text{if the bar was not used OR the leftover } Rl_j < W \end{cases}$$

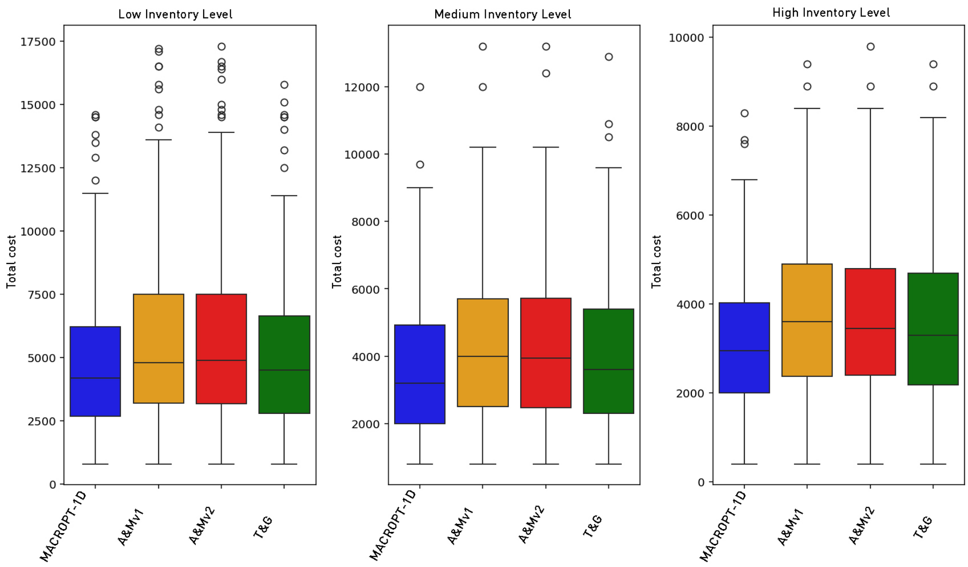

Figure 4. Comparison of total cost across models

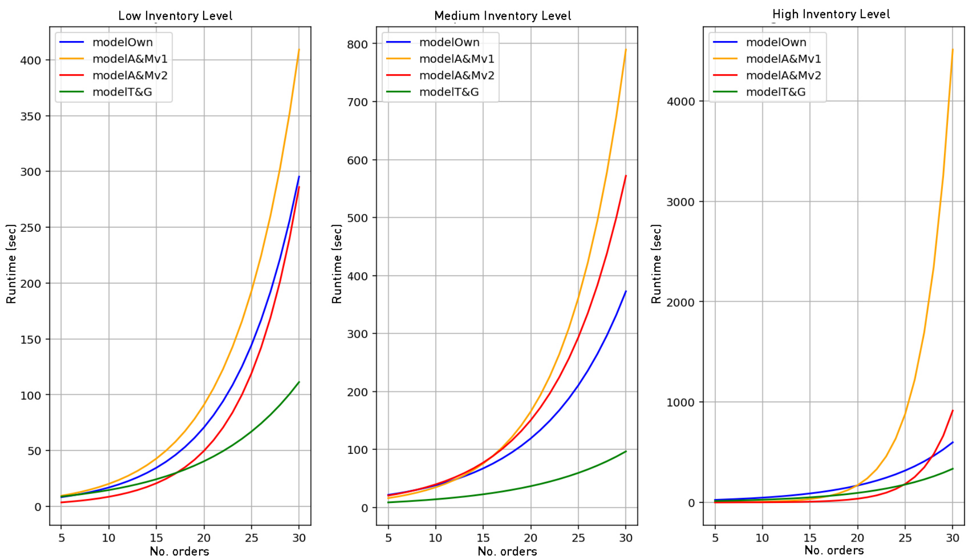

Figure 5. Comparison of runtime

Let us suppose we have an optimal solution in which all bars except j_0 were not returned to the stock, i.e.

$$\sum_{j=1|j\neq j_0}^{m} YRT_j = 0$$

and moreover, none of the orders was assigned to the bar j_0.

$$\sum_{i=1}^{n} x_{ij_0} = 0.$$

Denote the optimal value of the other variables by
$Wl^* = (Wl_1^*, Wl_2^*, \ldots, Wl_m^*)$,
$YU^* = (YU_1^*, YU_2^*, \ldots, YU_{j_0-1}^*, YU_{j_0+1}^*, \ldots, YU_m^*)$,
$YRT^* = (YRT_1^*, YRT_2^*, \ldots, YRT_{j_0-1}^*, YRT_{j_0+1}^*, \ldots, YRT_m^*)$.
In this case both of the values $YU_j = 0; YRT_j = 0$ and $YU_j = 1; YRT_j = 1$ are feasible and also optimal solutions for model (23) - (28)

Case $YU_{j_0} = 0; YRT_{j_0} = 0$

As bar j_0 has no orders assigned to it, $\sum_{i=1}^{n} x_{ij_0} ol_i = 0$. Constraints (26) and (27), that contain those variables are satisfied so

$$W \cdot 0 \ \leq \ bl_j \cdot 0 - 0 \ \leq \ Wl^*_{j_0} + 0 \cdot M.$$

Case $YU_{j_0} = 1; YRT_{j_0} = 1$

Beside the previous case, values $YU_{j_0} = 1; YRT_{j_0} = 1$ are also possible and satisfy the affected constraints, so

$$W \cdot 1 \ \leq \ bl_j \cdot 1 - 0 \ \leq \ Wl^*_{j_0} + 1 \cdot M.$$

Because the value of YU_{j_0} and YRT_{j_0} do not influence the objective value, this means both are optimal solutions.

This may be misleading information, as the new variable incorrectly indicates that the given bar is returned to inventory during the operation, even though it was not used at all to fulfill the orders. In other words it is possible YU_{j_0} and YRT_{j_0} to be 1 even though bar j_0 is not used, i.e. does not have any orders assigned to it $\sum_{i=1}^{n} x_{ij_0} ol_i = 0$.

An appropriate correction could be the following change of constraint (24) :

$$\sum_{i=1}^{n} x_{ij} ol_i + Wl_j = bl_j \cdot YU_j \qquad j = 1, 2, \ldots, m$$

So, this constraint and the objective (23) together will force variables YU_{j_0} and YRT_{j_0} in case of an unused bar j_0 to be 0, correctly.

Instead of changing the constraint (24) another possible way for correction is to introduce the following two restrictions:

$$\sum_{i=1}^{n} x_{ij} \geq YU_j \qquad\qquad j = 1, 2, \ldots, m$$

$$\sum_{i=1}^{n} x_{ij} \geq YRT_j \qquad\qquad j = 1, 2, \ldots, m$$

These constraints force the binary variables YU_j and YRT_j to be zeros if the bar j has no orders assigned to it, so it is not used neither returned to stock.

5. Summary

This paper addresses multi-objective one-dimensional cutting stock problems, with the aim of improving the efficiency of industrial resource utilization while reducing operational costs and material waste. The primary objective is the implementation and extensive simulation-based evaluation of a previously developed multi-objective cutting optimization model and three other models. MACROPT-1D is compared against three models published in the literature: two variants of the model introduced by Abuabara and Morabito [14], and the model proposed by Trkman and Gradisar [15].

All models were implemented in Python using the academic version of Gurobi Optimizer.

With respect to the first research objective outlined in Section 1.1, the comparative analysis is based on both a numerical example and a large-scale simulation study comprising 900 randomly generated test instances, structured into nine scenario groups along two independent dimensions: three inventory levels (low, medium, high) and three order-size scenarios.

Regarding the second objective, model performance was cross-evaluated with respect to five criteria simultaneously: the number of cuts generated, the amount of waste produced, the number of stock bars used, total cost (including cutting and waste penalties), and computational runtime, irrespective of which of these criteria forms part of a model's own native objective. This cross-evaluation revealed trade-offs that would have remained hidden under a single-criterion assessment. Most notably, the waste-only models of Abuabara and Morabito [14] were found to generate *more* waste, on average, than the multi-objective MACROPT-1D and T&G models, a consequence of the side constraint limiting the number of recycled remainders, which is intended to prevent inventory explosion but inadvertently penalizes waste minimization itself.

Concerning the third objective, the results show that the proposed multi-objective model MACROPT-1D, which simultaneously minimizes the number of cuts, waste, and the number of used stock bars, achieves superior performance in reducing the number of cuts and total cost, and in many cases also outperforms waste-focused models in waste minimization. This improved performance comes at the expense of increased raw material usage, as the proposed model tends to utilize a larger number of stock bars. In terms of computational efficiency, the T&G model proved to be the fastest, while the proposed model exhibited moderate running times.

Finally, with respect to the fourth objective, the theoretical correctness analysis identified an inconsistency in the A&Mv2 model. Simulation results revealed that in approximately 1.33% of the cases, the model may incorrectly classify a stock bar as both used and returned to inventory, despite no order being assigned to it. To address this issue, two additional linear constraints or an appropriate change of the original length balance constraint are proposed, restoring logical consistency and yielding correct results in subsequent tests.

5.1. Future research directions

The findings of this study suggest several promising directions for future work. First, the comparison presented here relies on a fixed set of cost coefficients and a single, weighted-sum scalarization of the competing objectives; an explicit Pareto-based analysis, using scalarization techniques such as the ε-constraint or Chebyshev methods, would allow the trade-offs between the number of cuts, waste, and the number of affected bars to be characterized more comprehensively than a single linear combination of weights permits.

Second, the runtime analysis in Section 3.6 indicates that all four exact MILP formulations face rapidly increasing computational demands as the number of orders grows; extending MACROPT-1D with a matheuristic or heuristic solution approach, in the spirit of recent column-generation- and local-branching-based methods for related cutting stock variants [12], could substantially improve scalability to industrial-size instances without sacrificing the model's multi-objective structure. Third, the present model and its competitors are formulated for a single planning period with deterministic order lengths and quantities; incorporating multiple planning periods or demand uncertainty, following recent developments in the broader cutting stock literature [7], would bring the model closer to real industrial practice, where orders typically arrive over time and are subject to variability. Fourth, extending the present one-dimensional framework to a two-dimensional setting, analogous to recent multi-objective formulations for sheet and plate cutting [9], would broaden the applicability of the proposed adjustable, parametric objective beyond bar-cutting industries.

Finally, validating MACROPT-1D on real industrial datasets, following the practice of recent applied studies, such as BIM-based rebar cutting optimization in construction [4], rather than relying solely on randomly generated instances, would provide further evidence of its practical applicability and help quantify the managerial trade-off between waste reduction and inventory heterogeneity observed in this study.

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Conflict of interest

There is no conflict of interest to disclose.

Author contributions

Anett Rácz: Conceptualization, Methodology, Supervision, Writing – original draft, Writing – review & editing.
Ákos Szabó: Software, Visualization, Writing – original draft, Writing – review & editing.

Declaration of using AI tools

The authors declare that they have not used any type of generative artificial intelligence for the conceptual writing of this manuscript, nor for the creation of images, graphics, tables, or their corresponding captions. AI tools were solely used to identify relevant recent literature for the literature review and for basic linguistic corrections. All AI-suggested sources were independently verified by the authors before being cited.

References

[1] A.A. Shahin and O.M. Salem. "Using genetic algorithms in solving the one-dimensional cutting stock problem in the construction industry." *Canadian Journal of Civil Engineering*, vol. 31, no. 2, 2004, pp. 321-332. `https://doi.org/10.1139/l03-101`

[2] D. Tanır, O. Ugurlu, A. Guler and U. Nuriyev. "One-dimensional cutting stock problem with divisible items: A case study in steel industry." *TWMS Journal of Applied and Engineering Mathematics*, vol. 9, no. 3, 2019, pp. 473-484.

[3] G.A. Ogunranti and A.E. Oluleye. "Minimizing waste (off-cuts) using cutting stock model: The case of one dimensional cutting stock problem in wood working industry." *Journal of Industrial Engineering and Management (JIEM)*, vol. 9, no. 3, 2016, pp. 834-859. `https://doi.org/10.3926/jiem.1653`

[4] Z. Wang, et al. "Minimization of rebar cutting waste using BIM and cutting pattern-oriented multiobjective optimization." *Journal of Construction Engineering and Management*, vol. 150, no. 11, 2024, 04024166. `https://doi.org/10.1061/JCEMD4.COENG-15104`

[5] B.S.C. Campello, et al. "A multiobjective integrated model for lot sizing and cutting stock problems." *Journal of the Operational Research Society*, vol. 71, no. 9, 2020, pp. 1466-1478. `https://doi.org/10.1080/01605682.2019.1619892`

[6] G.G. Guimarães, K.C. Poldi and M. Martin. "Mathematical models for the one-dimensional cutting stock problem with setups and open stacks." *Journal of Combinatorial Optimization*, vol. 49, no. 3, 2025, pp. 43. `https://doi.org/10.1007/s10878-025-01276-5`

[7] V. Senergues, N. Brahimi, A.C. Cherri, F. Klein and O. Péton. "Cutting stock problem with usable leftovers: A review." *European Journal of Operational Research*, vol. 328, no. 1, 2026, pp. 1-14. `https://doi.org/10.1016/j.ejor.2025.03.014`

[8] A.C. Cherri, M.N. Arenales, H.H. Yanasse, K.C. Poldi and A.C.G. Vianna. "The one-dimensional cutting stock problem with usable leftovers-A survey." *European Journal of Operational Research*, vol. 236, no. 2, 2014, pp. 395-402. `https://doi.org/10.1016/j.ejor.2013.11.026`

[9] Y. Ma, J. Zhang, X. Yang, J. Li, X. Su and H. Chen. "Two-dimensional cutting stock problem with flexible length and usable leftovers in the steel industry." *European Journal of Operational Research*, vol. 326, no. 2, 2025, pp. 207-219. `https://doi.org/10.1016/j.ejor.2025.04.036`

[10] L.J. Montiel-Arrieta, et al. "Minimizing the total waste in the one-dimensional cutting stock problem with the African buffalo optimization algorithm." *PeerJ Computer Science*, 9, 2023, e1728. `https://doi.org/10.7717/peerj-cs.1728`

[11] G.M. Bressan, M.H. Pimenta-Zanon and F. Sakuray. "A tree-based heuristic for the one-dimensional cutting stock problem optimization using leftovers." *Materials*, vol. 16, no. 22, 2023, 7133. `https://doi.org/10.3390/ma16227133`

[12] E. de Araújo Silva Oliveira, et al. "A local branching-based solution for the multi-period cutting stock problem with tardiness, earliness, and setup costs." *Journal of Heuristics*, vol. 31, no. 1, 2025, 16. https://doi.org/10.1007/s10732-025-09547-4

[13] A. Rácz. "A MILP model for one dimensional cutting stock problem with adjustable leftover threshold and cutting cost." *An International Journal of Optimization and Control: Theories & Applications*, vol. 15, no. 2, 2025, pp. 215-224. https://doi.org/10.36922/ijocta.1660

[14] A. Abuabara and R. Morabito. "Cutting optimization of structural tubes to build agricultural light aircrafts." *Annals of Operations Research*, vol. 169, no. 1, 2009, pp. 149-165. https://doi.org/10.1007/s10479-008-0438-7

[15] P. Trkman and M. Gradisar. "One-dimensional cutting stock optimization in consecutive time periods." *European Journal of Operational Research*, vol. 179, no. 2, 2007, pp. 291-301. https://doi.org/10.1016/j.ejor.2006.03.027