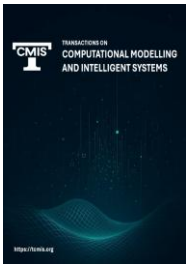

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RESEARCH ARTICLE

Conformable sliding mode control and analysis of Lorenz chaotic system

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Abstract

This study comparatively and innovatively examines linear state feedback control (LSFC) of the Conformable Sliding Mode Control (CSMC) methods on the conformable fractional-order Lorenz system. The extreme sensitivity of chaotic systems to initial conditions and their nonlinear nature present significant challenges for classical control methods. In this context, the conformable fractional derivative approach integrates the memory effect and fractional order character of the system into the model by adding a t^{q-1} multiplier to the system dynamics, while preserving the classical differential form. Thus, both analytical simplification is achieved and the physical interpretation of fractional dynamics is preserved. One of the original contributions of the study is the investigation of the conformable fractional-order form of the Lorenz system under control and the systematic comparison of two different control strategies under the same performance criteria. Innovatively, the effect of the conformable fractional-order on control performance is parametrically analysed, and the decisive role of the fractional order (q) on system stability and convergence speed is revealed. By activating the control signals after a specific delay, the natural evolution of the system in a chaotic regime was observed, and a realistic suppression scenario was created. This approach offers a more practical engineering perspective compared to instantaneous control applications. The results showed that the LSFC method provides a simple control structure and smooth control action. In contrast, the CSMC method achieves shorter settling time, faster convergence, improved disturbance rejection capability, and higher robustness against external disturbances and parameter uncertainties.

Keywords: Conformable, fractional, sliding mode, Lorenz, chaotic system

1. Introduction

The concept of conformable fractional-order derivatives has been developed as an alternative to classical fractional derivative definitions (such as Riemann–Liouville and Caputo) and has become a noteworthy approach in recent years, particularly in the field of control systems. Defined by reference, the conformable derivative largely preserves the classical structure of the differential operator while imparting fractional-order behaviour to the system via a t^{q-1} multiplier. This feature simplifies analytical operations and makes it possible to integrate the memory effect of fractional systems into control design [1].

Conformable fractional calculus has been extensively investigated from both theoretical and numerical perspectives. New mathematical properties of conformable fractional derivatives have been reported in the literature, further strengthening their analytical foundation [2], [3]. Moreover, conformable finite-element formulations have been developed for fractional partial differential equations, demonstrating the applicability of the conformable framework to large-scale engineering problems [4]. Recent studies have also addressed the stability analysis of conformable infinite-dimensional systems [5], semilinear systems [6] and conformable Θ-H systems [7], highlighting the growing importance of conformable fractional calculus in modern control theory and dynamical systems analysis.

Although not directly classified by this name in traditional control literature, conformable fractional differential systems are addressed within the scope of controllability and model-based feedback design. In this context, Wang and Fečkan provided a theoretical basis for model-based control design by examining the controllability conditions of conformable differential systems [8]. Similarly, Ding et al. investigated conformable impulsive differential equations by establishing solution representations and analyzing asymptotic as well as generalized Ulam–Hyers–Rassias stability [9]. Furthermore, Sadek contributed to model-based control approaches by examining the stability and observability conditions in conformable linear dynamic systems [10]. In linear state feedback control, the main goal is to cancel out nonlinear terms through feedback and transform the fault dynamics into a stable structure under the assumption that the system dynamics are known. Lyapunov-based analyses are generally used in such studies, and the effect of the fractional degree parameter (q) on system performance is examined [8]. Current research shows that the fractional degree can be used not only as a mathematical extension but also as an adjustable design parameter on control energy and settling time. CSMC has a more application-oriented and broader scope in the literature. Its robust structure and high performance under parameter uncertainties and external disturbances have led to its adaptation to conformable fractional systems. Alfi demonstrated the stability of conformable fractional chaotic systems by designing a sliding mode control system using the Lyapunov approach [11]. This work is significant because it shows how a conformable derivative operator can be used in sliding surface design. Conformable versions of terminal sliding mode control are also found in the literature. Haghighatnia designed a conformable fractional terminal sliding mode controller for uncertain nonlinear systems and analysed its finite-time convergence property [12]. Furthermore, Muñoz-Vázquez et al. proposed a generalized conformable sliding mode control approach, redefining the classical sliding surface with a conformable derivative structure and presenting their stability analyses in detail [13]. When these studies are evaluated, it is seen that conformable sliding mode control is extensively investigated, especially in the suppression of chaotic systems, stabilization of uncertain nonlinear systems, and finite-time control problems [11], [12]. The fractional-order parameter can affect both the convergence speed and the robustness level by shaping the dynamics of the sliding surface [13].

In this paper, the authors propose an approach that simplifies this task by creating novel sufficient conditions that allow the Lyapunov function candidate to increase in certain regions when traditional Lyapunov-based stability analysis is not feasible. The study examines equilibrium point stability, asymptotic stability, and global asymptotic stability [14]. Generally, the literature shows that conformable traditional control stands out with its model accuracy and low energy consumption, while conformable sliding mode control provides high robustness, fast convergence, and resilience to uncertainties [15]; the fractional order parameter is used as a decisive design parameter in both methods, influencing the behaviour of the control signal and system stability. A review of the literature reveals that studies on conformable linear state feedback control are more focused on theoretical controllability and model-based design, while studies on conformable sliding mode control are more intensively addressed with a focus on application and robustness. The linear state feedback control approach generally stands out with its simple implementation and smooth control signal generation, while conformable sliding mode control provides faster convergence and resilience to parameter uncertainties. Current studies demonstrate that the fractional order can be used as an important design parameter in both methods.

2. Mathematical modelling and formulation

2.1 Sliding mode control

SMC is a widely used method for controlling nonlinear systems due to its high robustness against uncertainties and external disturbances. The classical SMC approach aims to achieve asymptotic stability by orienting and holding system states on a predefined sliding surface. The main advantage of this method is that when the sliding mode is reached, the system dynamics become independent of parameter uncertainties and coupled disturbances. The fundamental principles of SMC were first systematically formulated by Vadim Utkin [16]. In sliding mode control, a sliding surface is primarily defined. System states hit this sliding surface, causing them to return to the origin. In this state, the system becomes stable. Sliding mode control consists of two phases reaching and sliding phases. In classical sliding mode control, high-frequency oscillations (chattering) occur on the sliding surface. In the control of dynamic systems such as electric motors, this chattering causes the motor to run erratically or jerkily, thus damaging the device. To eliminate this harmful effect, a bounded layer is created around the sliding surface. In the literature, functions such as fuzzy logic, tanh, and saturation are used for this purpose [17]. In this study, the saturation function was used instead of the signum function used in classical sliding mode structure as shown in Figure 1. Sliding mode control design consists of two phases: sliding surface design and controller design. The sliding condition forces the system states to reach the sliding surface (reaching phase) and sends them back to the origin via the surface, holding them at the equilibrium point.

In the article, the authors propose a novel piecewise continuous system by introducing a threshold level for states in Lorenz and Rabinovich systems. They used sliding mode control to achieve adaptive synchronization between switched dynamic systems with unknown parameters [18]. This article presents the design of an adaptive terminal sliding mode controller for dynamic systems. The proposed sliding surface has finite-time convergence capability. The gains of the access law are adjusted as a function of the sliding surface. The control law has been applied to

quad copter unmanned aerial vehicles, and it has been observed that the controller is robust against external disturbances and parametric uncertainties [19].

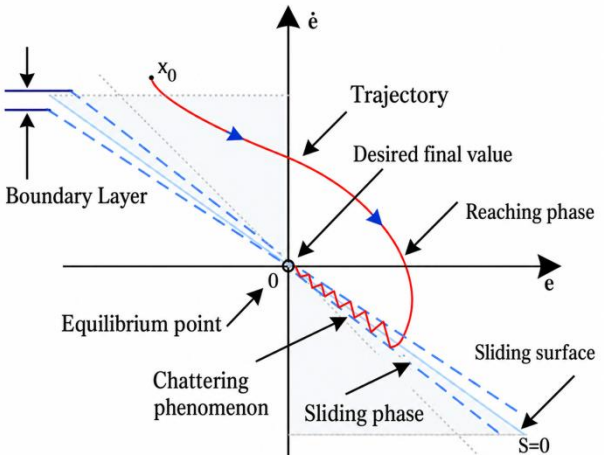


Figure 1. Sliding mode structure

2.2 Lorenz system equations

The Lorenz system is a system of nonlinear differential equations developed by Edward Lorenz in 1963 to simplify the model of atmospheric convection [20]. It played a fundamental role in the birth of chaos theory and demonstrated the possibility of unpredictable behaviour in deterministic systems. The system consists of three related nonlinear differential equations as below [20]:

$$\begin{aligned} \dot{x}_1 &= \sigma(x_2 - x_1) \\ \dot{x}_2 &= x_1(\rho - x_3) - x_2 \\ \dot{x}_3 &= x_1 x_2 - \beta x_3 \end{aligned} \tag{1}$$

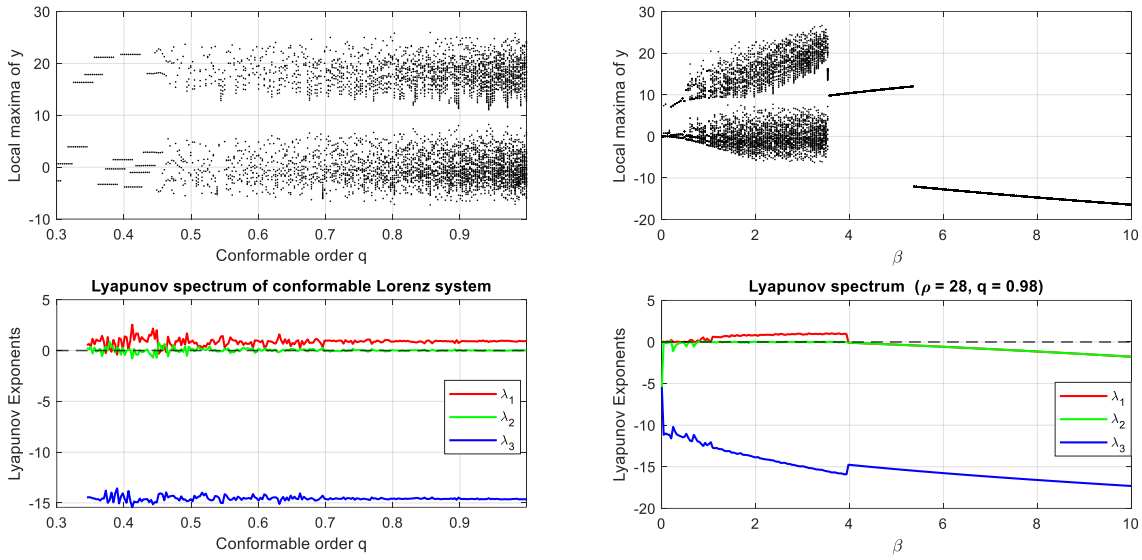
When system parameters set to $\sigma = 10$, $\rho = 28$, $\beta = 8/3$, Lorenz system produces chaotic flows. On the other hand, the conformable fractional-order Lorenz system is a generalization of the classical Lorenz system using the concept of fractional derivatives. The classical Lorenz system, developed by Edward Lorenz in 1963 to model atmospheric convection movements, is one of the best-known examples of chaotic behaviour. The system consists of three nonlinear differential equations and produces chaotic (irregular but deterministic) solutions at certain parameter values. In fractional order systems, the derivative order does not have to be an integer (such as 1, 2, 3); it can be a fractional value such as $0 < q \leq 1$ [21]. A conformable derivative is a definition of a fractional derivative that possesses properties similar to the classical derivative (linearity, multiplication rule, etc.) and is easier to analyse. Therefore, it is frequently preferred in engineering and physics problems. In conformable fractional derivatives, q-order conformable derivatives are used instead of first-order time derivatives in Lorenz equations.

For a differentiable function, the conformable fractional derivative compatible with order is defined as follows [12]:

$$T^q x(t) = t^{1-q} \frac{dx(t)}{dt}, 0 < q \leq 1 \tag{2}$$

Using the conformable derivative definition $T^q x(t) = t^{1-q} \frac{dx(t)}{dt}$, the ordinary derivative can be expressed as $\frac{dx(t)}{dt} = t^{q-1} T^q x(t)$. Therefore, a conformable fractional-order system of the form $T^q x = f(x)$ can be equivalently rewritten as $\dot{x} = t^{q-1} f(x)$. This equivalent representation enables the conformable system to be simulated using conventional ODE solvers while preserving the influence of the conformable fractional-order parameter (q). Using this formulation, the classical Lorenz system given in Eq. (2) can be rewritten as [20]:

$$\begin{aligned} T^q x_1 &= \sigma(x_2 - x_1) \\ T^q x_2 &= x_1(\rho - x_3) - x_2 \\ T^q x_3 &= x_1 x_2 - \beta x_3 \end{aligned} \tag{3}$$



(a) (b)

Figure 2. Bifurcation diagram and Lyapunov spectrum of conformable fractional-order Lorenz chaotic system (a) $\sigma = 10$, $\rho = 28$ and $\beta = 8/3$, (b) $\sigma = 10$, $\rho = 28$ and $q = 0.98$.

To investigate the influence of the conformable fractional-order parameter on the system dynamics, bifurcation diagrams and Lyapunov exponent spectra were generated with respect to both q and β, as shown in Figure 2. Figure 2(a) demonstrates that the system exhibits a transition from periodic to chaotic behavior as the conformable order q increases. The corresponding Lyapunov spectrum confirms this observation, where the largest Lyapunov exponent becomes positive for $q \geq 0.4$, indicating the onset of chaos. Moreover, the chaotic regime persists over a wide range of q values, including the selected operating point q=0.98. Figure 2(b) presents the bifurcation and Lyapunov analyses with respect to the parameter β while keeping q=0.98. It can be observed that the system maintains chaotic dynamics for (0< β <4), where the largest Lyapunov exponent remains positive. These results verify that the chosen parameter set $\sigma = 10$, $\rho = 28$, $q = 0.98$ lies within a well-developed chaotic region and is therefore suitable for the subsequent control studies. For given parameters values, the system exhibits chaotic behaviour.

2.1. Control law design

Feedback linearization-based state feedback control is employed as a benchmark controller in this study. The control law given in Eq. (4) is designed to cancel the nonlinear dynamics of the Lorenz system and impose stable linear closed-loop dynamics.

$$u(t) = -f(x) - Kx \tag{4}$$

where K>0 is the feedback gain matrix. The conformable Lorenz system is represented as

$$\dot{x} = t^{q-1}\left(f(x) + Bu\right) \tag{5}$$

where $B = \begin{bmatrix}1 & 0 & 0\end{bmatrix}^T$ is the input distribution vector, indicating that the control signal is applied through the first state equation. Substituting Eq. (4) into the system dynamics yields:

$$\begin{aligned} \dot{x} &= t^{q-1}\left(f(x) - f(x) - Kx\right) \\ \dot{x} &= t^{q-1}(-Kx) \end{aligned} \tag{6}$$

Therefore, the nonlinear dynamics are cancelled and the resulting closed-loop system behaves as a conformable linear state-feedback system. As the control input is applied through the first state equation, the controller directly regulates the x_1 state, while the x_2 and x_3 states are influenced indirectly through the inherent coupling of the Lorenz dynamics.

Fractional-order systems produce more realistic results in many physical and engineering systems due to their ability to model memory and hereditary properties compared to classical integer-order models. In this context, the

conformable derivative, which is among the fractional derivative definitions, offers significant advantages in control design because it both preserves the local structure and has chain rule and linearity properties similar to the classical derivative.

CSMC is achieved by adapting the classical SMC structure to conformable fractional-order systems. In this approach, system dynamics are modelled using a conformable derivative operator, and the sliding surface is usually defined as a linear combination:

$$s = cx \tag{7}$$

Error vector of system can be written as below:

$$e(t) = x(t) - x_{ref} \tag{8}$$

Since the goal is for the states to go directly to zero, the x_{ref} value is taken as zero. In this case, the error is written as follows.

$$e_1(t) = x_1(t), \quad e_2(t) = x_2(t), \quad e_3(t) = x_3(t) \tag{9}$$

c is a suitably chosen constant coefficient vector in sliding mode. The aim is to arrange the conformable derivative of the sliding variable such that it is negative definite. In this respect, a typical reaching law can be defined as follows:

$$c(f(x) + Bu) = -ks - \eta sat(s/\phi) \tag{10}$$

Here, k>0 represents the control gain, boundary layer thickness parameter is ϕ and $sat(\cdot)$ represents the saturation function. Increasing ϕ enlarges the boundary layer around the sliding surface, which reduces switching activity and mitigates the chattering phenomenon. However, an excessively large ϕ may decrease tracking accuracy and increase the steady-state error [22]. Using the saturation function reduces the chattering effect compared to the classical sign function. If the control signal 'u' in the equation is left alone:

$$u = -\frac{cf(x) + ks + \eta sat(s/\phi)}{cB} \tag{11}$$

Since the system is single-input, the input distribution vector is selected as $B = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T$ which results in $cB = 1$. The derivative of sliding surface can be written as below:

$$\dot{s} = c\dot{x} = t^{q-1}(-ks - \eta sat(s/\phi) \tag{12}$$

To verify the stability of the proposed conformable sliding mode controller, consider the Lyapunov candidate function

$$V(s) = \frac{1}{2}s^2 \tag{13}$$

Taking the time derivative yields dot $V = s\dot{s}$. Substituting Eq. (12) gives

$$\dot{V} = s, t^{q-1}\left(-ks - \eta, sat\left(\frac{s}{\phi}\right)\right) \tag{14}$$

Thus,

$$-t^{q-1}\left(ks^2 + \eta, s, sat\left(\frac{s}{\phi}\right)\right) \tag{15}$$

Since ($0<q\leq 1$), it follows that ($t^{q-1}>0$) for (t>0). Moreover, $[s, sat\left(\frac{s}{\phi}\right) \geq 0.]$ and $\dot{V} \leq -t^{q-1}ks^2 \leq 0.$ Hence, $V(s)$ is a non-increasing function and the sliding variable converges toward the sliding surface. Consequently, the proposed conformable sliding mode control law guarantees asymptotic stability of the closed-loop system.

The control applies pressure not only to x_1 but also to x_2 and x_3 via the combination selected with c, forcing them onto the sliding surface simultaneously (c=[1 0.8 0.5]). Depending on the sliding surface c values, it can be written as follows:

$$s = x_1 + 0.8x_2 + 0.5x_3 \tag{16}$$

In both methods, the conformable part is used in the same location.

$$\dot{x} = t^{q-1}(...) \tag{17}$$

The most important advantage of SMC in the conformable framework is that it offers lower computational complexity compared to classical fractional derivative definitions (e.g., Riemann–Liouville or Caputo) and can be easily integrated with standard numerical solution methods thanks to its time-scaling structure. Therefore, it stands out as an effective and practical method for stabilizing chaotic systems.

Especially in strongly nonlinear and conditionally sensitive systems like the Lorenz System, CSMC enables the systematic convergence of the sliding variable to zero, allowing all state variables to be guided to a stable equilibrium point in a coordinated manner. Furthermore, its invariance against coupled disturbances allows for faster convergence and higher robustness performance compared to traditional control methods.

The superior disturbance rejection capability of the proposed CSMC method can be explained by the structure of the sliding surface. In the LSFC approach, the control input is applied through a single channel and directly regulates only the x_1 state, while the x_2 and x_3 states are influenced indirectly through the inherent coupling of the Lorenz dynamics. Consequently, disturbances acting on the system may propagate through the uncontrolled states and lead to larger oscillations. In contrast, the proposed CSMC method employs a sliding surface of the form $s = cx$, where all state variables participate simultaneously in the control objective. Therefore, deviations in any state variable contribute to the sliding variable and are actively compensated by the controller. This coordinated state regulation improves disturbance attenuation, accelerates convergence, and enhances the overall robustness of the closed-loop system.

3. Simulation studies

In this study, the Lorenz chaotic system was mathematically modeled using the conformable derivative definition, and the system's control performance was analyzed through numerical simulations performed in MATLAB. The system equations were coded in MATLAB, and both the LSFC method and the proposed CSMC method were applied comparatively under the same initial conditions and disturbance scenarios. In the simulations, the robustness performance of the control methods was evaluated by applying the disturbance to the system at a specific time point. The results obtained are presented graphically, and the changes in state variables over time are examined. The coefficient values, are used in the conformable sliding mode controller in the simulation studies were taken as k=6, η=10, ϕ=0,05. Lorenz parameters were taken as $x(0)$=[1,5 -2 3] and and q=0.98. Both controllers become active after 20 seconds. The proposed CSMC operates faster and goes to zero compared to the LSFC.

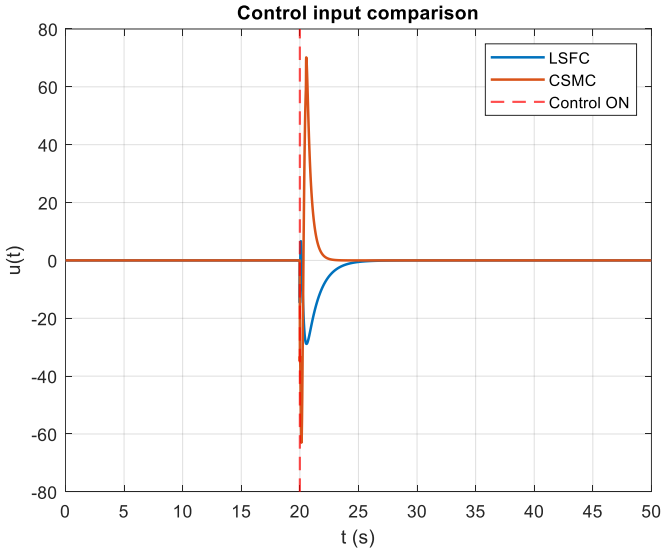


Figure 3. Comparison of LSFC and CSMC control input signals after controller activation.

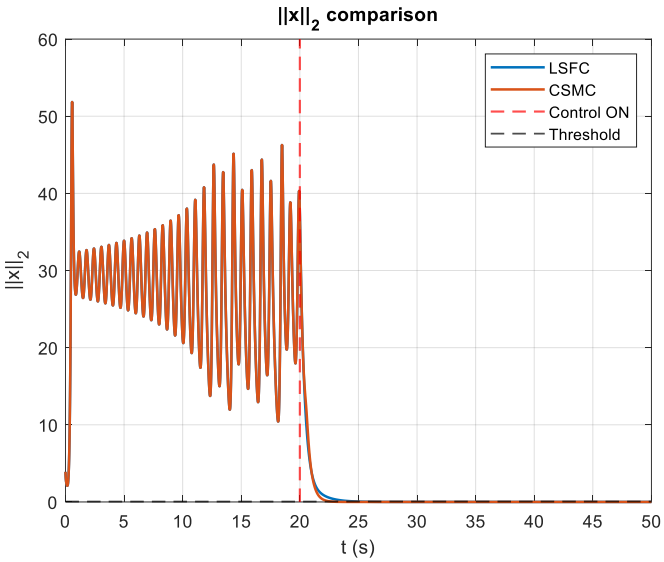


Figure 4. Comparison of the Euclidean norm of the state vector ||x||₂ for the LSFC and CSMC methods when the control action is activated at t=20 s.

Figure 4 shows that the CSMC method drives the state norm to zero significantly faster than the LSFC method. This result demonstrates the superior convergence performance of the proposed controller.

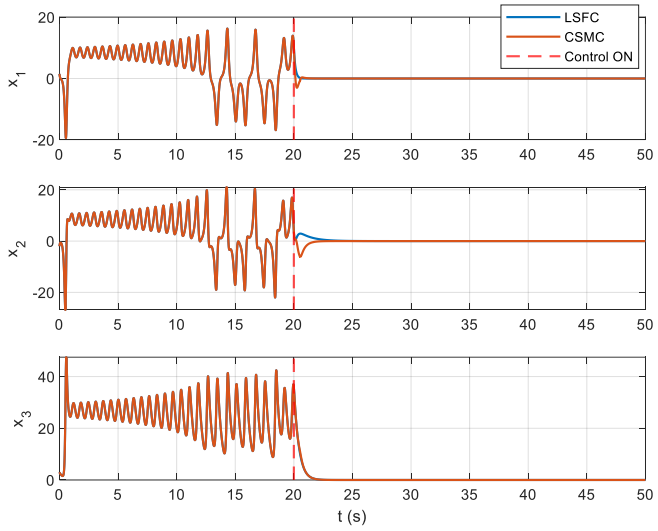


Figure 5. State trajectories of the conformable fractional-order Lorenz system under LSFC and CSMC.

Looking at the state trajectories given in Figure 5, it can be seen that x_1 and x_2 settle at approximately the same time (equilibrium point), while the CSMC settles at zero earlier in x_3.

4. Comparative analysis of two controllers

To measure the system's response to disturbances, a variable signal was added to the circuit after 20 seconds. The mathematical expression for the added complex signal is given below.

$$d(t)=\begin{cases}0, & t<t_{on}\\ 8\sin(4t)+6\cos(2t)+4\sin(0.5t) & t>t_{on}\end{cases} \tag{17}$$

The disturbance signal is activated in 20 seconds along with the controller. The change in the disturbance signal over time is shown in Figure 6. To ensure accurate comparison, the same disturbance was added to both controllers for the same duration.

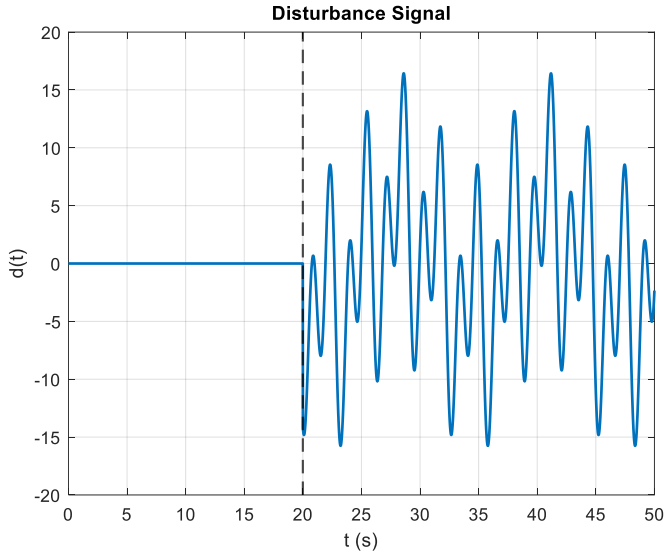


Figure 6. External disturbance signal applied to both controllers after t = 20 s.

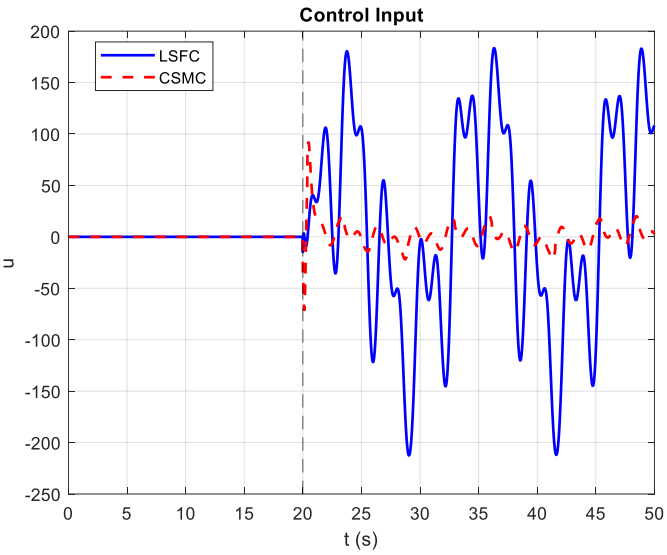


Figure 7. Comparison of LSFC and CSMC control signals under external disturbance conditions. Disturbance and control are activated at t=20 s.

Figure 7 compares the control signals generated by the LSFC and CSMC methods under external disturbance conditions. Before the activation instant ((t<20) s), the control inputs remain zero. After the controllers and disturbance are activated at (t=20) s, both methods generate control actions to suppress the chaotic dynamics. It can be observed that the LSFC method requires significantly larger control effort, resulting in high-amplitude control oscillations. In contrast, the CSMC method produces a more bounded control signal while maintaining effective stabilization performance. These results indicate that the proposed CSMC approach achieves robust disturbance rejection with a lower control amplitude and smoother control behaviour under the considered operating conditions.

Figure 8 compares the Euclidean norm of the state x_1, for the LSFC and CSMC methods under external disturbance conditions. During the first 20 seconds, when no control action is applied, the state norm exhibits oscillatory behaviour due to the chaotic nature of the Lorenz system. After the controllers are activated at (t=20) s, the CSMC method rapidly drives the state norm toward zero and maintains it close to the equilibrium point despite the presence of disturbances. In contrast, the LSFC method reduces the state norm but fails to completely suppress the oscillations, resulting in persistent fluctuations. These results indicate that the proposed CSMC approach provides faster convergence, improved disturbance rejection capability, and enhanced robustness compared with the LSFC method.

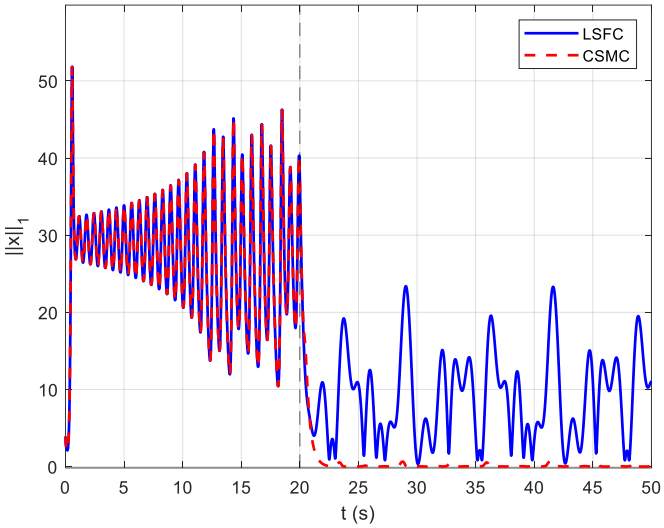


Figure 8. Comparison of the Euclidean norm of the state x_1 under external disturbance for the LSFC and CSMC methods.

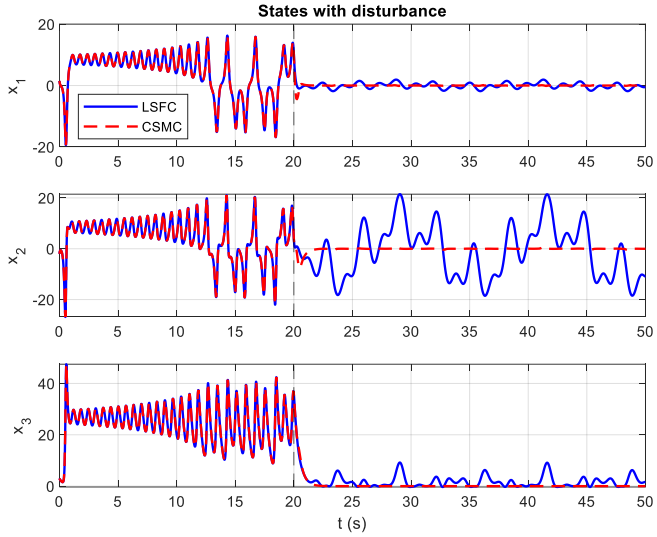


Figure 9. Comparison of state trajectories under external disturbance for LSFC and CSMC methods.

Figure 9 presents the performance of LSFC and CSMC methods for three state variables (x_1, x_2, x_3) of a Lorenz system under external disturbance. Up to approximately 20 seconds after the control is activated, increasing amplitude oscillations originating from the chaotic structure are observed in all state variables. After the disturbance is applied, the LSFC method reduces the oscillation amplitudes but cannot completely suppress the disturbance effects, particularly in the x_2 and x_3 states. In contrast, the CSMC method rapidly attenuates the oscillations and maintains all state variables close to the equilibrium point, demonstrating superior robustness against external disturbances. It is particularly noteworthy that CSMC provides both faster transient regime performance and lower steady-state error. These results demonstrate that the conformable sliding mode control method offers higher stability, stronger external disturbance suppression, and superior robust control performance for all state variables of chaotic systems under external disturbance. When all the results obtained are evaluated together, it is seen that the Conformable Sliding Mode Control method offers much higher robustness, faster convergence, lower oscillation amplitude, and superior stability compared to the LSFC method in controlling nonlinear and chaotic systems such as Lorenz under disturbing effects.

In the LSFC method, the control rule uses only linear feedback. In this approach, only the x_1 state is directly suppressed via a single input channel. x_2 and x_3 are indirectly affected. In the case of a chaotic Lorentz system, when the control is activated, x_1 is rapidly damped, while the deviations of the other states may decrease more slowly. This method does not have a chattering problem, and the control inputs are smooth. However, model accuracy is critical for high performance; the stabilization and accuracy of the system can be seriously affected in the presence of parameter deviations or measurement noise. In conformable sliding mode control, all states are

oriented together across the sliding surface. The control law includes both a linear and a switching term. Thanks to this structure, the surface dynamics work in accordance with the conformable derivative factor, and coordinated damping of all states is ensured. In addition, this control strategy is more robust against parameter uncertainty and external disturbances, i.e., it offers robust control characteristics. Although there is an inherent risk of chattering in the control, these effects can be greatly reduced with the saturation function and boundary layer.

In the literature, SMC in single-input chaotic systems is mostly performed with linearization + classical sliding surface. In this study, stabilization of a wider state space with a single input is achieved by controlling the combination of x_1, x_2, x_3 with the sliding surface cx form. This contributes to the coordinated damping of states, especially in chaotic systems. CSMC gains robustness thanks to its reaching and switching terms. It provides orientation to the sliding surface even under external disturbance or parameter deviation. In most studies in the literature, LSFC or classical SMC is used in chaotic systems; in this study, robust conformable SMC is used. The control domain can be adjusted through the sliding surface cx parameters; this allows the system to be optimized in different chaotic regimes. Combining SMC with the conformable fractional-order approach, controlling the single-input chaotic Lorenz system in a multi-state manner, and simultaneously evaluating performance and robustness adds originality to the study. Its contribution to the literature is that it presents a simulation-verified method for stabilizing chaotic systems with a robust and flexible control strategy.

5. Conclusions and future research

This study investigates the performance of CSMC method in controlling a Lorenz chaotic system under disturbing influences, and compares the obtained results with the LSFC method. The extreme sensitivity of chaotic systems to initial conditions, their nonlinear structure, and their susceptibility to disturbing influences make the control of such systems quite difficult and critical. Therefore, the development of control methods with high robustness, fast convergence, and effective disturbance suppression is of great importance. In this context, the system stability, transient regime performance, and behaviour under disturbing influence of the proposed conformable sliding mode control approach were analysed, and the control effectiveness was comprehensively evaluated. The findings show that the proposed method offers significant advantages in controlling chaotic systems. In this study, the performance of the LSFC method and the CSMC method were comprehensively compared for the x_1, x_2, and x_3 state variables of the Lorenz chaotic system under disturbing influences.

Examination of the figures reveals that in the first 20 seconds before control is activated, all state variables exhibit irregular and increasing amplitude oscillations, consistent with the chaotic system characteristics. This clearly demonstrates that the system's inherent structure is unstable and sensitive to initial conditions. However, significant performance differences emerged between the two control methods from the moment control was activated. While the application of the LSFC method resulted in a partial improvement in system behaviour, it was observed that high-amplitude oscillations persisted, particularly in state variables x_1 and x_2, and the system struggled to converge precisely to the zero equilibrium point. Furthermore, oscillations could not be completely suppressed in state variable x_3, and persistent fluctuations continued. Although the LSFC method improves system behaviour after controller activation, its disturbance rejection capability remains limited compared with the proposed CSMC approach. The CSMC method quickly converged all state variables to the zero reference shortly after control activation and significantly suppressed oscillations. It is clearly observed that the transient regime duration is shorter, the maximum overshoot values are lower, and the steady-state error is reduced to a negligible level. Furthermore, it is understood that the system's stability is effectively maintained despite disturbing effects, and chaotic behaviour is successfully eliminated. When all the results are evaluated together, it is clearly demonstrated that the Conformable Sliding Mode Control method provides much higher robustness, faster convergence, lower oscillation amplitude, and superior stability performance compared to LSFC methods in controlling nonlinear and chaotic systems such as Lorenz under disturbing effects. These results confirm that the CSMC method is a highly effective and reliable control approach for controlling chaotic systems. It is anticipated that future studies can achieve better results by using optimization methods to find the optimum values for controller parameters and boundary layer width. In addition, the applicability of the proposed conformable sliding mode control framework to practical nonlinear systems, such as fractional-order SIR epidemic models, can be investigated to further demonstrate its real-world utility.

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Conflict of interest

There is no conflict of interest to disclose.

Author contributions

All authors contributed equally and approved the final manuscript.

Declaration of using AI tools

The authors declare that they have not used any type of generative artificial intelligence for the writing of this manuscript, nor for the creation of images, graphics, tables, or their corresponding captions.

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